Math 140 Exam #1 Key (Spring 2024)

1a Factoring the numerator, we have

$$\lim_{t\to -1} \frac{[(2t-1)-3][(2t-1)+3]}{t+1} = \lim_{t\to -1} 2(2t-4) = 2[2(-1)-4] = -12.$$

1b Getting a common denominator, we have

$$\lim_{x\to 2} \frac{x-2}{x(x-2)} = \lim_{x\to 2} \frac{1}{x} = \frac{1}{2}.$$

1c Multiply by the conjugate of the numerator:

$$\lim_{x\to 3} \left(\frac{\sqrt{x+1}-2}{x-3} \cdot \frac{\sqrt{x+1}+2}{\sqrt{x+1}+2} \right) = \lim_{x\to 3} \frac{x-3}{(x-3)(\sqrt{x+1}+2)} = \lim_{x\to 3} \frac{1}{\sqrt{x+1}+2} = \frac{1}{4}.$$

1d Factoring the denominator:

$$\lim_{\theta\to 0} \frac{\cos\theta - 1}{(\cos\theta - 1)(\cos\theta + 1)} = \lim_{\theta\to 0} \frac{1}{\cos\theta + 1} = \frac{1}{2}.$$

2a $\displaystyle \lim_{y\to 1} \frac{2y}{|1-y|} = \frac{2}{(\to 0^+)} = +\infty$

2b $\displaystyle \lim_{z\to 3^-} \frac{(z-2)(z-1)}{z-3} = \frac{(1)(2)}{(\to 0^-)} = -\infty$

2c $\displaystyle \lim_{x\to 2^+} \frac{1}{\sqrt{x(x-2)}} = \frac{1}{\sqrt{2(\to 0^+)}} = \frac{1}{(\to 0^+)} = +\infty$

2d $\displaystyle \lim_{x\to 2^-} \frac{1}{\sqrt{x(x-2)}} = \frac{1}{\sqrt{2(\to 0^-)}} = \frac{1}{\sqrt{(\to 0^-)}} =$ DNE, since the square root of a negative number is not real-valued.

2e $\displaystyle \lim_{x\to -\infty} (3x^7 + x^2) = \lim_{x\to -\infty} x^2(3x^5+1) = (\to -\infty)^2(\to -\infty) = (+\infty)(-\infty) = -\infty$

3a Since $\sqrt{x^2} = |x|$ in general, we have

$$\lim_{x\to\infty} \frac{|x|\sqrt{1+2/x+6/x^2}}{x-1} = \lim_{x\to\infty} \frac{x\sqrt{1+2/x+6/x^2}}{x-1} = \lim_{x\to\infty} \frac{\sqrt{1+2/x+6/x^2}}{1-1/x} = \frac{\sqrt{1}-0}{1-0} = 1,$$

and

$$\lim_{x\to-\infty}\frac{|x|\sqrt{1+2/x+6/x^2}}{x-1}=\lim_{x\to-\infty}\frac{-x\sqrt{1+2/x+6/x^2}}{x-1}=\lim_{x\to\infty}\frac{-\sqrt{1+2/x+6/x^2}}{1-1/x}=-1,$$

and so the horizontal asymptotes are $y=1$ and $y=-1$.

3b The only candidate for a vertical asymptote is $x=1$, but we must see if $\lim_{x\to 1^-} f(x)$ or $\lim_{x\to 1^+} f(x)$ are infinite. In fact,

$$\lim_{x\to 1}\left(\frac{\sqrt{x^2+2x+6}-3}{x-1}\cdot\frac{\sqrt{x^2+2x+6}+3}{\sqrt{x^2+2x+6}+3}\right)=\lim_{x\to 1}\frac{x+3}{\sqrt{x^2+2x+6}+3}=\frac{2}{3}\neq\pm\infty,$$

and so $x=1$ is *not* a vertical asymptote. There is no vertical asymptote.

4a $\lim\limits_{x\to 0^+} F(x)=\lim\limits_{x\to 0^+} 2x^3=0\neq 1=F(0)$, so that $\lim\limits_{x\to 0} F(x)\neq F(0)$, and thus F is not continuous at 0.

4b We already found that $\lim_{x\to 0^+} F(x)\neq F(0)$, so F is not continuous from the right at 0; but

$$\lim_{x\to 0^-} F(x)=\lim_{x\to 0^-}(x^3+4x+1)=1=F(0)$$

shows that F is continuous from the left at 0.

5a We have

$$g'(2)=\lim_{h\to 0}\frac{g(2+h)-g(2)}{h}=\lim_{h\to 0}\frac{\dfrac{1}{3h+5}-\dfrac{1}{5}}{h}=\lim_{h\to 0}\frac{5-(3h+5)}{5h(3h+5)}=\lim_{h\to 0}\frac{-3}{5(3h+5)}=-\frac{3}{25}.$$

5b Slope of the tangent line at $(2,\frac{1}{5})$ is $g'(2)=-\frac{3}{25}$, so the line is $y=-\frac{3}{25}x+\frac{11}{25}$.