Math 140 Exam #1 Key (Spring 2021)

1 We have

$$\lim_{x\to -1} F(x) = 3, \quad \lim_{x\to -2} F(x) = 1, \quad \lim_{x\to 1^-} F(x) = 2, \quad \lim_{x\to 1^+} F(x) = 4, \quad \lim_{x\to 3^+} F(x) = \text{DNE}.$$

2a Simply reduce the fraction first:

$$\lim_{x\to -b} \frac{(x+b)^6 + (x+b)^9}{4} = 0.$$

2b Multiply top and bottom of fraction by $5(5+h)$:

$$\lim_{h\to 0} \frac{5-(5+h)}{5h(5+h} = \lim_{h\to 0} \frac{-1}{5(5+h)} = -\frac{1}{25}.$$

2c Factor the bottom (or multiply top and bottom by $\sqrt{x}+8$):

$$\lim_{x\to 64} \frac{\sqrt{x}-8}{(\sqrt{x}-8)(\sqrt{x}+8)} = \lim_{x\to 64} \frac{1}{\sqrt{x}+8} = \frac{1}{\sqrt{64}+8} = \frac{1}{16}.$$

2d Factor the bottom:

$$\lim_{x\to 0} \frac{1-\cos x}{(\cos x - 2)(\cos x - 1)} = \lim_{x\to 0} \frac{-1}{\cos x - 2} = \frac{-1}{\cos 0 - 2} = 1.$$

3 Since

$$\lim_{x\to -3^-} G(x) = \lim_{x\to -3^-} (3x - 4k) = -9 - 4k \quad \text{and} \quad \lim_{x\to -3^+} G(x) = \lim_{x\to -3^+} (x+9) = 6,$$

the limit $\lim_{x\to -3} G(x)$ can only exist if $-9-4k = 6$, which only happens if $k = -\frac{15}{4}$. Then $\lim_{x\to -3} G(x) = 6$.

4 All equal $-\infty$.

5 Since

$$f(x) = \frac{x+1}{x(x-2)^2},$$

f has vertical asymptotes $x=0$ and $x=2$. We find that

$$\lim_{x\to 0^-} f(x) = -\infty, \quad \lim_{x\to 0^+} f(x) = +\infty, \quad \lim_{x\to 2^-} f(x) = +\infty, \quad \lim_{x\to 2^+} f(x) = +\infty,$$

so $\lim_{x\to 2} f(x) = +\infty$ and $\lim_{x\to 0} f(x)$ does not exist.

6 Divide top and bottom by x^2, then evaluate to get $-\frac{4}{7}$.

7 In general,

$$f(x) = \frac{4x^3+1}{2x^3+\sqrt{16x^6+1}} = \frac{4x^3+1}{2x^3+|x|^3\sqrt{16+x^{-6}}},$$

so

$$\lim_{x\to\infty} f(x) = \lim_{x\to\infty} \frac{4x^3+1}{2x^3+x^3\sqrt{16+x^{-6}}} = \lim_{x\to\infty} \frac{4+x^{-3}}{2+\sqrt{16+x^{-6}}} = \frac{4}{2+\sqrt{16}} = \frac{2}{3},$$

and

$$\lim_{x\to-\infty} f(x) = \lim_{x\to-\infty} \frac{4x^3+1}{2x^3-x^3\sqrt{16+x^{-6}}} = \lim_{x\to-\infty} \frac{4+x^{-3}}{2-\sqrt{16+x^{-6}}} = \frac{4}{2-\sqrt{16}} = -2.$$

Horizontal asymptotes are $y = \frac{2}{3}$ and $y = -2$.

8 Continuity from the left at -1 requires that $\lim_{x\to-1^-} h(x) = h(-1)$. Since

$$\lim_{x\to-1^-} h(x) = \lim_{x\to-1^-} (2x^2 - x) = 3,$$

and $h(-1) = s$, we set $s = 3$ to get continuity from the left at -1.

Continuity from the right at -1 requires $\lim_{x\to-1^+} h(x) = h(-1)$. Since

$$\lim_{x\to-1^+} h(x) = \lim_{x\to-1^+} (3x - 5) = -8,$$

and $h(-1) = s$, we set $s = -8$ to get continuity from the right at -1.

9a By definition,

$$f'(1) = \lim_{t\to1} \frac{f(t)-f(1)}{t-1} = \lim_{t\to1} \frac{\sqrt{2t-1}-1}{t-1} = \lim_{t\to1} \frac{2t-2}{(t-1)\left(\sqrt{2t-1}+1\right)} = \lim_{t\to1} \frac{2}{\sqrt{2t-1}+1} = 1.$$

9b Slope of the tangent line at $(1,1)$ is $f'(1) = 1$, so line is $y = x$.

10 We have

$$f'(x) = \lim_{t\to x} \frac{f(t)-f(x)}{t-x} = \lim_{t\to x} \frac{\dfrac{1}{3t-4} - \dfrac{1}{3x-4}}{t-x} = \lim_{t\to x} \frac{-3(t-x)}{(t-x)(3t-4)(3x-4)}$$

$$= \lim_{t\to x} \frac{-3}{(3t-4)(3x-4)} = -\frac{3}{(3x-4)^2}.$$