MATH 140 EXAM #4 KEY (FALL 2020)

1 We have

$$y = \sqrt{8x - x^2} \quad \Rightarrow \quad y^2 = 8x - x^2 \quad \Rightarrow \quad (x^2 - 8x) + y^2 = 0 \quad \Rightarrow \quad (x-4)^2 + y^2 = 16,$$

so the integral represents the area of the top left quarter of a circular disc centered at $(4, 0)$ with radius 4, and so equals 4π.

2a $\displaystyle\int_1^4 (x^{1/2} - 2x^{-1/2})\,dx = \left[\frac{2}{3}x^{3/2} - 4x^{1/2}\right]_1^4 = \frac{2}{3}.$

2b Integral equals $\big[-\cot\theta\big]_{\pi/4}^{\pi/2} = 1.$

3 By the Fundamental Theorem of Calculus:

$$\frac{d}{dx}\int_{x^2}^0 \frac{t}{\sin^3 t + 9}\,dt = -\frac{d}{dx}\int_0^{x^2} \frac{t}{\sin^3 t + 9}\,dt = -\frac{x^2}{\sin^3(x^2) + 9} \cdot \frac{d}{dx}(x^2) = -\frac{2x^3}{\sin^3(x^2) + 9}.$$

4a Let $u = 4 - 9x^2$ to get

$$\int \frac{-1/8}{\sqrt{u}}\,du = -\frac{1}{18}(2u^{1/2}) + C = -\frac{\sqrt{4 - 9x^2}}{9} + C.$$

4b Let $u = \cos x$, so:

$$\int \frac{\sin x}{\cos^8 x}\,dx = -\int \frac{1}{u^8}\,du = \frac{1}{7}u^{-7} + C = \frac{1}{7}\sec^7 x + C.$$

4c Let $u = 16 - x^4$, so integral becomes:

$$-\int_{16}^0 \frac{\sqrt{u}}{2}\,du = -\frac{1}{2}\left[\frac{2}{3}u^{3/2}\right]_{16}^0 = \frac{64}{3}.$$

5 Area is

$$A = \int_0^2 \big[(3x - x^2) - x\big]\,dx + \int_2^3 \big[x - (3x - x^2)\big]\,dx = \frac{8}{3}.$$

6 Volume is

$$V = \int_0^3 A(x)\,dx = \int_0^3 \frac{1}{2}\pi\left(\frac{3-x}{2}\right)^2 dx = \frac{\pi}{8}\int_0^3 (x^2 - 6x + 9)\,dx = \frac{9\pi}{8}.$$

7 Disc Method here:

$$V = \int_2^4 \pi\left(\sqrt{25 - x^2}\right)^2 dx = \frac{94}{3}\pi.$$

8 Making the substitution $u = x^2$, volume is

$$V = \int_0^{\sqrt{\pi/2}} 2\pi x \cos(x^2)\, dx = \int_0^{\pi/4} \pi \cos u\, du = \pi\left[\sin u\right]_0^{\pi/4} = \frac{\pi}{\sqrt{2}}.$$