MATH 140 EXAM #2 KEY (FALL 2020)

1a $s'(t) = -20t^9 - 7 - \dfrac{8}{t^2}$.

1b Since $f(x) = x^{1/2}(x^2 - 1) = x^{5/2} - x^{1/2}$, we have $f'(x) = \frac{5}{2}x^{3/2} - \frac{1}{2}x^{-1/2}$.

1c $g'(r) = \dfrac{(1+\sqrt{r})(2r) - r^2\left(\dfrac{1}{2\sqrt{r}}\right)}{(1+\sqrt{r})^2} = \dfrac{2r + 2r^{3/2} - \frac{1}{2}r^{3/2}}{(1+\sqrt{r})^2} = \dfrac{4r + 3r\sqrt{r}}{2(1+\sqrt{r})^2}$.

1d $h'(\theta) = \theta^2 \sec\theta \tan\theta + 2\theta \sec\theta - \cos\theta$.

1e $y' = \dfrac{1}{3}(1 + 2x - x^5)^{-2/3} \cdot (2 - 5x^4)$.

1f $y' = -8x\sec^2(\cos 4x^2)\sin 4x^2$.

2 With the Quotient Rule,

$$f'(x) = \frac{2x^2 - 10x + 6}{(3 - x^2)^2} \quad \Rightarrow \quad f''(x) = \frac{4x^3 - 30x^2 + 36x - 30}{(3 - x^2)^3}.$$

3 Let $f(x) = x + \sqrt{x}$, and find x such that $f'(x) = 2$, or $1 + \frac{1}{2\sqrt{x}} = 2$, which has solution $x = \frac{1}{4}$. Thus the tangent line has slope 2 and point $(\frac{1}{4}, f(\frac{1}{4})) = (\frac{1}{4}, \frac{3}{4})$. Equation for line is thus $y = 2x + \frac{1}{4}$.

4 Implicit differentiation yields

$$\frac{1}{2}(x^4 + y^2)^{-1/2}(4x^3 + 2yy') = 10 - 3y^2y',$$

where of course y' is dy/dx. The rest is simply algebra:

$$\frac{dy}{dx} = \frac{10\sqrt{x^4 + y^2} - 2x^3}{3y^2\sqrt{x^4 + y^2} + y}.$$

5 When $x = 2$ equation becomes $2y^2 = 32$, and so $y = \pm 4$. So there are two relevant points on the curve: $(2, \pm 4)$. Implicit differentiation of the original equation gives

$$12x^2 = 2yy'(4 - x) - y^2 \quad \Rightarrow \quad \frac{dy}{dx} = \frac{12x^2 + y^2}{2y(4 - x)}.$$

At $(2, 4)$: $y' = \frac{64}{16} = 4$, so tangent line is $y = 4x - 4$. At $(2, -4)$: $y' = \frac{64}{-16} = -4$, so tangent line is $y = -4x + 4$.

6a Set $h'(t) = 0$ to get $-9.8t + 19.6 = 0$, and thus $t = 2$. At 2 seconds the stone reaches its maximum height, which is $h(2) = 44.1$ ft.

6b Solve $h(t) = 0$ to get $t = 5$, so the stone hits the ground after 5 seconds, and its velocity is $h'(5) = -9.8(5) + 19.6 = -29.4$ ft/s.

7 The snowball loses mass, or volume V, at a rate proportional to its surface area S: $\dfrac{dV}{dt} = kS$, where k is a constant of proportionality. Now,

$$\frac{dV}{dt} = kS \;\Rightarrow\; \frac{d}{dt}\left(\frac{4}{3}\pi r^3\right) = k(4\pi r^2) \;\Rightarrow\; 4\pi r^2 \cdot \frac{dr}{dt} = k(4\pi r^2),$$

and finally

$$\frac{dr}{dt} = k.$$

8 Let x be the kite's distance to the north from Esther. Let ℓ be the length of string from hand to kite. We have a right triangle with legs length x and 60 and hypotenuse length ℓ. Thus $x^2 + 60^2 = \ell^2$. Now, $dx/dt = 8$, so

$$x^2 + 60^2 = \ell^2 \;\Rightarrow\; 2x\frac{dx}{dt} = 2\ell\frac{d\ell}{dt} \;\Rightarrow\; \frac{d\ell}{dt} = \frac{8x}{\ell} = \frac{8}{\ell}\sqrt{\ell^2 - 60^2}.$$

Hence when $\ell = 130$ we get

$$\left.\frac{d\ell}{dt}\right|_{\ell=130} = \frac{8}{130}\sqrt{130^2 - 60^2} \approx 7.1 \text{ ft/s}.$$