Math 140 Exam #1 Key (Fall 2020)

1a Simply reduce the fraction first:

$$\lim_{x\to c}\frac{(x-c)^2}{x-c}=\lim_{x\to c}(x-c)=c-c=0.$$

1b Rationalize the numerator and reduce:

$$\lim_{y\to 3}\left(\frac{\sqrt{3y+16}-5}{y-3}\cdot\frac{\sqrt{3y+16}+5}{\sqrt{3y+16}+5}\right)=\lim_{y\to 3}\frac{3}{\sqrt{3y+16}+5}=\frac{3}{10}.$$

1c Factor and reduce:

$$\lim_{\theta\to\pi/2}\frac{(\sin\theta-4)(\sin\theta-1)}{(\sin\theta+1)(\sin\theta-1)}=\lim_{\theta\to\pi/2}\frac{\sin\theta-4}{\sin\theta+1}=\frac{1-4}{1+1}=-\frac{3}{2}.$$

1d Combine the fractions:

$$\lim_{r\to 2}\frac{r-2}{r(r-2)}=\lim_{r\to 2}\frac{1}{r}=\frac{1}{2}.$$

2 Since

$$\lim_{x\to -1^-}q(x)=\lim_{x\to -1^-}(x^2-5x)=6\quad\text{and}\quad\lim_{x\to -1^+}q(x)=\lim_{x\to -1^+}(2\ell x^3-7)=-2\ell-7,$$

the limit $\lim_{x\to -1}q(x)$ can only exist if $-2\ell-7=6$, which only happens if $\ell=-\frac{13}{2}$. Then $\lim_{x\to -1}q(x)=6$.

3 The fraction in the limit, which I'll call $f(t)$, reduces to $\dfrac{1}{t^3(t+2)}$, which helps to determine that

$$\lim_{t\to 0}f(t)=\text{DNE},\quad \lim_{t\to 2}f(t)=\frac{1}{32},\quad \lim_{t\to -2}f(t)=\text{DNE}.$$

4 For $x\to\infty$ we have $\sqrt{x^2}=|x|=x$, and for $x\to-\infty$ we have $\sqrt{x^2}=|x|=-x$. This results in the horizontal asymptotes $y=\pm\frac{1}{3}$:

$$\lim_{x\to\infty}U(x)=\lim_{x\to\infty}\frac{x+1}{x\sqrt{9+1/x}}=\lim_{x\to\infty}\frac{1+1/x}{\sqrt{9+1/x}}=\frac{1}{\sqrt{9}}=\frac{1}{3}.$$

and

$$\lim_{x\to-\infty}U(x)=\lim_{x\to-\infty}\frac{x+1}{-x\sqrt{9+1/x}}=\lim_{x\to-\infty}\frac{1+1/x}{-\sqrt{9+1/x}}=-\frac{1}{\sqrt{9}}=-\frac{1}{3}.$$

5 For $x < 0$, $F(x) = x^3 + 4x + 1$, so F is continuous on $(-\infty, 0)$ since polynomial functions are continuous on their domains. Similarly, for $x > 0$, $F(x) = 2x^3$, so F is continuous on $(0, \infty)$ also. However,

$$\lim_{x \to 0^+} F(x) = \lim_{x \to 0^+} 2x^3 = 0 \neq 1 = F(0)$$

and

$$\lim_{x \to 0^-} F(x) = \lim_{x \to 0^-} (x^3 + 4x + 1) = 1 = F(0),$$

so F is continuous from the left at 0, but not from the right. Therefore F is continuous on $(-\infty, 0]$ and $(0, \infty)$.

6a $$f'(1) = \lim_{x \to 1} \frac{f(x) - f(1)}{x - 1} = \lim_{x \to 1} \frac{\dfrac{x-1}{x+2} - 0}{x - 1} = \lim_{x \to 1} \frac{1}{x+2} = \frac{1}{3}.$$

6b The line has point $(1, f(1)) = (1, 0)$ and slope $\frac{1}{3}$, so equation is $y = \frac{1}{3}x - \frac{1}{3}$.

7 We have

$$\begin{aligned} v'(t) &= \lim_{x \to t} \frac{v(x) - v(t)}{x - t} = \lim_{x \to t} \left(\frac{\sqrt{2 - 4x} - \sqrt{2 - 4t}}{x - t} \cdot \frac{\sqrt{2 - 4x} + \sqrt{2 - 4t}}{\sqrt{2 - 4x} + \sqrt{2 - 4t}} \right) \\ &= \lim_{x \to t} \frac{4(t - x)}{(x - t)(\sqrt{2 - 4x} + \sqrt{2 - 4t})} = -4 \lim_{x \to t} \frac{1}{\sqrt{2 - 4x} + \sqrt{2 - 4t}} \\ &= -\frac{2}{\sqrt{2 - 4t}}. \end{aligned}$$