MATH 140 EXAM #4 KEY (FALL 2015)

1a $\int_1^7 f + \int_7^9 f = \int_1^9 f$ implies $\int_1^7 f = \int_1^9 f - \int_7^9 f = -1 - 5 = -6$.

1b We have

$$\int_9^7 (2h - f) = -\int_7^9 (2h - f) = \int_7^9 (f - 2h) = \int_7^9 f - 2\int_7^9 h = 5 - 2(4) = -3.$$

2 We have

$$y = \sqrt{24 - 2x - x^2} \quad \Rightarrow \quad y^2 = 24 - 2x - x^2 \quad \Rightarrow \quad (x^2 + 2x) + y^2 = 24 \quad \Rightarrow \quad (x+1)^2 + y^2 = 25,$$

so the integral represents the area of the top half of a circular disc centered at $(-1, 0)$ with radius 5:

$$\int_{-6}^{4} \sqrt{24 - 2x - x^2}\,dx = \frac{1}{2}\pi(5)^2 = \frac{25}{2}\pi.$$

3a $\displaystyle\int_{1/2}^{1} (x^{-3} - 8)\,dx = \left[-\tfrac{1}{2}x^{-2} - 8x\right]_{1/2}^{1} = -\tfrac{5}{2}$.

3b $\displaystyle\int_0^{\pi/8} 3\sin(2x)\,dx = \left[-\tfrac{3}{2}\cos(2x)\right]_0^{\pi/8} = -\tfrac{3}{2}(\cos\tfrac{\pi}{4} - \cos 0) = -\tfrac{3}{2}\left(\tfrac{1}{\sqrt{2}} - 1\right) = \tfrac{3}{2} - \tfrac{3}{2\sqrt{2}}$

4 By the Fundamental Theorem of Calculus,

$$\frac{d}{dx}\int_2^{x^5} \frac{4}{t^3}\,dt = \frac{4}{(x^5)^3} \cdot (x^5)' = \frac{4}{x^{15}} \cdot 5x^4 = \frac{20}{x^{11}}.$$

5a Let $u = x^4 + 16$, so $x^3\,dx$ becomes $\frac{1}{4}du$, and we get

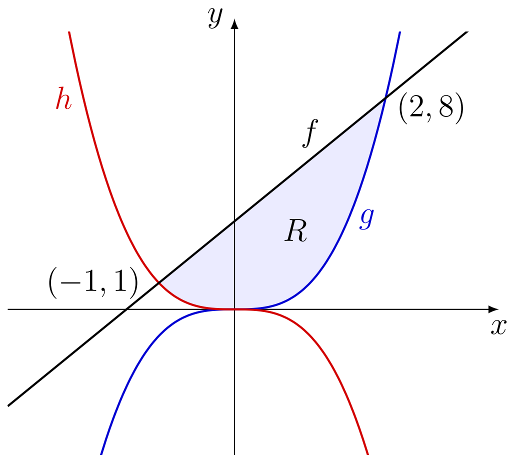
$$\int x^3(x^4 + 16)^6\,dx = \frac{1}{4}\int u^6\,du = \frac{1}{4} \cdot \frac{1}{7}u^7 + C = \frac{1}{28}(x^4 + 16)^7 + C.$$

5b Let $u = y + 1$, so $y^2 = (u-1)^2$, and dy becomes du. Then

$$\int \frac{y^2}{(y+1)^4}\,dy = \int \frac{(u-1)^2}{u^4}\,du = \int \left(\frac{1}{u^2} - \frac{2}{u^3} + \frac{1}{u^4}\right) du = -\frac{1}{u} + \frac{1}{u^2} - \frac{1}{3u^3} + C$$

$$= -\frac{1}{y+1} + \frac{1}{(y+1)^2} - \frac{1}{3(y+1)^3} + C.$$

6 Let $f(x) = \frac{7}{3}x + \frac{10}{3}$, $g(x) = x^3$, and $h(x) = -x^3$. The enclosed region R is shown below.



We have

$$\mathcal{A}(R) = \int_{-1}^{0}(f-h) + \int_{0}^{2}(f-g) = \int_{-1}^{0}\big(x^3 + \tfrac{7}{3}x + \tfrac{10}{3}\big)\,dx + \int_{0}^{2}\big(-x^3 + \tfrac{7}{3}x + \tfrac{10}{3}\big)\,dx$$
$$= \tfrac{23}{12} + \tfrac{22}{3} = \tfrac{37}{4}.$$

7 Volume is

$$V = \int_0^2 A(x)\,dx = \int_0^2 \frac{1}{2}\pi\left(\frac{2-x}{2}\right)^2 dx = \frac{\pi}{8}\int_0^2 (x^2 - 4x + 4)\,dx = \frac{\pi}{3}.$$

8 Volume is

$$V = \int_1^4 \pi\left(\frac{1}{x^2}\right)^2 dx = \pi\int_1^4 x^{-4}\,dx = \frac{21\pi}{64}.$$

9 For $f(x) = \frac{1}{3}(x^2+2)^{3/2}$, length of curve is

$$L = \int_1^2 \sqrt{1 + [f'(x)]^2}\,dx = \int_1^2 \sqrt{1 + \left[\tfrac{1}{2}(x^2+2)^{1/2}(2x)\right]^2}\,dx$$
$$= \int_1^2 \sqrt{x^4 + 2x^2 + 1}\,dx = \int_1^2 \sqrt{(x^2+1)^2}\,dx = \int_1^2 (x^2+1)\,dx$$
$$= \left[\tfrac{1}{3}x^3 + x\right]_1^2 = \tfrac{10}{3}.$$