

1a Since $\sqrt{t} = t^{1/2}$, $s'(t) = 2t^{-1/2} - t^3 + 1$.

1b Quotient Rule:

$$f'(x) = \frac{(x^2 - 1)(4x^3) - (x^4 + 1)(2x)}{(x^2 - 1)^2} = \frac{4x^5 - 4x^3 - 2x^5 - 2x}{(x^2 - 1)^2} = \frac{2x^5 - 4x^3 - 2x}{(x^2 - 1)^2}.$$

1c Product Rule:

$$y' = \sin x \sec^2 x + \cos x \tan x = \tan x \sec x + \sin x.$$

1d Quotient Rule:

$$y' = \frac{(1 + \sin x)(-2 \sin x) - (2 \cos x)(\cos x)}{(1 + \sin x)^2} = -\frac{2 + 2 \sin x}{(1 + \sin x)^2} = -\frac{2}{1 + \sin x}.$$

2 First we get $f'(x) = 6x^2 - 6x - 12$. Setting $f'(x) = 60$ gives $6x^2 - 6x - 12 = 60$, or $x^2 - x - 12 = 0$, which factors as $(x - 4)(x + 3) = 0$. Solutions are $x = -3, 4$. Points on the graph of f with slope 60 are therefore $(-3, -41)$ and $(4, 36)$.

3a $y' = 16(4 - 15x^4)(4x - 3x^5)^{15}.$

3b $y' = \sec^2(\sqrt{x}) \cdot (\sqrt{x})' = \frac{1}{2\sqrt{x}} \sec^2 \sqrt{x}.$

3c $h'(x) = 4 \sin^3(\cos 7x) \cdot \cos(\cos 7x) \cdot (-\sin 7x) \cdot 7 = -28 \cos(\cos 7x) \sin^3(\cos 7x) \sin 7x.$

4 By the Quotient Rule:

$$3x^2 = \frac{(x - y)(1 + y') - (x + y)(1 - y')}{(x - y)^2} \Rightarrow y' = \frac{3x^2(x - y)^2 + 2y}{2x}.$$

Alternative: rewrite equation as $x^4 - x^3y = x + y$ to avoid having to use the Quotient Rule, if desired. Now differentiate with respect to x :

$$4x^3 - 3x^2y - x^3y' = 1 + y' \Rightarrow 4x^3 - 3x^2y - 1 = x^3y' + y' = (x^3 + 1)y' \Rightarrow y' = \frac{4x^3 - 3x^2y - 1}{x^3 + 1}.$$

5 Implicit differentiation gives

$$y^{5/2} + \frac{5}{2}xy^{3/2}y' + \frac{3}{2}x^{1/2}y + x^{3/2}y' = 0,$$

and so

$$y' = -\frac{y^{5/2} + \frac{3}{2}x^{1/2}y}{x^{3/2} + \frac{5}{2}xy^{3/2}} = -\frac{2y^{5/2} + 3x^{1/2}y}{2x^{3/2} + 5xy^{3/2}}$$

At the point $(x, y) = (4, 1)$ we have

$$y' = -\frac{2(1)^{5/2} + 3(4)^{1/2}(1)}{2(4)^{3/2} + 5(4)(1)^{3/2}} = -\frac{2 + 3(2)}{2(8) + 5(4)} = -\frac{8}{25},$$

which is the slope of the tangent line. Equation of tangent line is therefore

$$y - 1 = -\frac{8}{25}(x - 4) \Rightarrow 25y - 25 = -8x + 32 \Rightarrow 8x + 25y = 57.$$

6 We differentiate the formula¹ $A = \frac{1}{2}bh$ with respect to time t to obtain

$$\frac{dA}{dt} = \frac{1}{2} \left(b \frac{dh}{dt} + h \frac{db}{dt} \right). \quad (1)$$

When area is $A = 150 \text{ cm}^2$ and height is $h = 12 \text{ cm}$ we find the base to be $b = 2A/h = 25 \text{ cm}$. Putting all known quantities into (1) gives

$$2 = \frac{1}{2} \left(25(-1) + 12 \frac{db}{dt} \right),$$

or $db/dt = 2\frac{5}{12} \text{ cm/min}$.

7 The ground, wall, and ladder form a right triangle with hypotenuse of length 13. If x is the distance between the wall and the foot of the ladder, and y is the distance between the ground and the top of the ladder, then $x^2 + y^2 = 13^2$. Both x and y are functions of time t , and so differentiating with respect to t yields

$$2x(t)x'(t) + 2y(t)y'(t) = 0 \Rightarrow y'(t) = -\frac{x(t)x'(t)}{y(t)}.$$

We're given that $x'(t) = 0.5 \text{ ft/s}$ for all $t \geq 0$, and of course we also have $y(t) = \sqrt{169 - x^2(t)}$. Thus

$$y'(t) = -\frac{x(t)}{2\sqrt{169 - x^2(t)}}.$$

Now, at the time t when $x(t) = 5 \text{ ft}$, we obtain

$$y'(t) = -\frac{5}{2\sqrt{169 - 5^2}} = -\frac{5}{24};$$

that is, at the time t when the foot of the ladder is 5 ft from the wall, the top of the ladder is sliding down the wall at a rate of $-5/24 \text{ ft/s}$.

¹Yes, in a calculus course you are expected to know how to find the area of a triangle.