MATH 140
FALL 2020
EXAM 4

NAME:

1. [10 pts.] Use geometry, and not Riemann sums, to evaluate the definite integral

$$\int_0^4 \sqrt{8x - x^2}\, dx.$$

(Hint: try completing the square.)

2. [10 pts. each] Evaluate each definite integral using the Fundamental Theorem of Calculus.

(a) $\displaystyle\int_1^4 \frac{x-2}{\sqrt{x}}\, dx$

(b) $\displaystyle\int_{\pi/4}^{\pi/2} \csc^2\theta\, d\theta$

3. [10 pts.] Find the derivative: $\displaystyle\frac{d}{dx}\int_{x^2}^0 \frac{t}{\sin^3 t + 9}\, dt.$

4. [10 pts. each] Use a change of variables (substitution) to find the following.

(a) $\displaystyle\int \frac{x}{\sqrt{4-9x^2}}\, dx$

(b) $\displaystyle\int \sin x \sec^8 x\, dx$

(c) $\displaystyle\int_0^2 x^3\sqrt{16-x^4}\, dx$

5. [10 pts.] Find the area of the region enclosed by the curves $y = 3x - x^2$, $y = x$, and $x = 3$.

6. [10 pts.] Use the General Slicing Method to find the volume of the solid whose base is the triangle with vertices $(0,0)$, $(3,0)$, and $(0,3)$, and whose cross sections perpendicular to the base and parallel to the y-axis are semicircles.

7. [10 pts.] Find the volume of the solid generated by revolving about the x-axis the region bounded by the curves $y = \sqrt{25 - x^2}$, $y = 0$, $x = 2$, and $x = 4$.

8. [10 pts.] Use cylindrical shells to find the volume of the solid generated when the region bounded by the curves $y = \cos(x^2)$, $x = 0$, $x = \frac{1}{2}\sqrt{\pi}$, and $y = 0$ is revolved about the y-axis.