MATH 140
FALL 2020 NAME:
EXAM 3

1. [10 pts.] Use the Closed Interval Method to find the absolute extreme values of

$$u(t) = \frac{\sin t \cos t - \sin t}{2}$$

on $[0, 2\pi]$.

2. [10 pts.] Show that the equation $x^3 + 25x + 8 = 7x^2$ has exactly one real solution.

3. [5 pts. each] Let $f(x) = \dfrac{10x^2}{x^2 + 3}$.
   (a) Find the domain, intercepts, and asymptotes of f.
   (b) Find f'.
   (c) Use the Monotonicity Test to find intervals of increase and decrease, and then use the First Derivative Test to find all local extrema.
   (d) Find f''.
   (e) Use the Concavity Test to find intervals of concavity for f, identifying all inflection points.

4. [10 pts.] Use optimization to find what point on the line $y = 4x - 6$ is closest to the origin.

5. [15 pts.] Among all the right circular cones with a slant height of 5, what are the dimensions (radius and height) that maximize the volume of the cone? The slant height of a cone is the distance from the outer edge of the base to the vertex. (Volume of cone formula is $V = \frac{1}{3}\pi r^2 h$.)

6. [10 pts.] Use linear approximation to approximate the value of $\sqrt[3]{65}$.

7. [10 pts. each] Use L'Hôpital's Rule to evaluate each limit.
   (a) $\displaystyle\lim_{x \to \infty} \frac{3}{x} \csc \frac{5}{x}$.
   (b) $\displaystyle\lim_{x \to 0^+} (\sin x) \sqrt{\frac{1 - x}{x}}$.