MATH 140
FALL 2020
EXAM 2

NAME:

1. [10 pts. each] Use differentiation rules to find the derivative of each function.

   (a) $s(t) = -2t^{10} - 7t + \frac{8}{t}$.

   (b) $f(x) = \sqrt{x}(x^2 - 1)$

   (c) $g(r) = \dfrac{r^2}{1 + \sqrt{r}}$

   (d) $h(\theta) = \theta^2 \sec\theta - \sin\theta$

   (e) $y = \sqrt[3]{1 + 2x - x^5}$

   (f) $y = \tan(\cos 4x^2)$

2. [10 pts.] Given that

$$f(x) = \frac{2x - 5}{3 - x^2},$$

find the second derivative $f''(x)$.

3. [10 pts.] Find the equation of the tangent line to the curve $y = x + \sqrt{x}$ that has slope 2.

4. [10 pts.] Use implicit differentiation to find dy/dx, given that $\sqrt{x^4 + y^2} = 10x - y^3$.

5. [10 pts.] Find the equations of *both* tangent lines to the curve given by $4x^3 = y^2(4 - x)$ at points on the curve where $x = 2$.

6. [10 pts. each] A stone is thrown vertically upward from the edge of a cliff on Earth with an initial velocity of 19.6 m/s from a height of 24.5 m above the ground. The height (in meters) of the stone above the ground t seconds after it's thrown is $h(t) = -4.9t^2 + 19.6t + 24.5$.

   (a) When does the stone reach its highest point? What is the height at that time? Round answers to the nearest tenth.

   (b) When does the stone strike the ground, and with what velocity? Round answers to the nearest tenth.

7. [10 pts.] A spherical snowball melts at a rate proportional to its surface area. Show that the rate of change of the radius is constant. (A sphere of radius r has volume $V = \frac{4}{3}\pi r^3$ and surface area $S = 4\pi r^2$.)

8. [15 pts.] Upon reaching a height of 60 ft above her hands, Esther's kite rises no higher but drifts due north in a wind blowing 8 ft/s. How fast is the string running through Esther's hands at the moment when she has released 130 ft of string?