

MATH 140  
 FALL 2020  
 EXAM 1

NAME:

1. [10 pts. each] Evaluate each limit algebraically using limit laws, showing work.

(a)  $\lim_{x \rightarrow c} \frac{x^2 - 2cx + c^2}{x - c}$ ,  $c$  a constant

(b)  $\lim_{y \rightarrow 3} \frac{\sqrt{3y+16} - 5}{y - 3}$

(c)  $\lim_{\theta \rightarrow \pi/2} \frac{\sin^2 \theta - 5 \sin \theta + 4}{\sin^2 \theta - 1}$

(d)  $\lim_{r \rightarrow 2} \left( \frac{1}{r-2} - \frac{2}{r^2 - 2r} \right)$

2. [10 pts.] Suppose

$$q(x) = \begin{cases} x^2 - 5x, & x \leq -1 \\ 2\ell x^3 - 7, & x > -1. \end{cases}$$

Determine a value for  $\ell$  (if any) for which the limit  $\lim_{x \rightarrow -1} q(x)$  exists, and state the value of the limit if possible.

3. [3 pts. each] Determine the following limits as a real number or  $\pm\infty$ , or state the limit does not exist.

$$\lim_{t \rightarrow 0} \frac{t-2}{t^5 - 4t^3}, \quad \lim_{t \rightarrow 2} \frac{t-2}{t^5 - 4t^3}, \quad \lim_{t \rightarrow -2} \frac{t-2}{t^5 - 4t^3}.$$

4. [15 pts.] Determine  $\lim_{x \rightarrow \infty} U(x)$  and  $\lim_{x \rightarrow -\infty} U(x)$  for

$$U(x) = \frac{x+1}{\sqrt{9x^2+x}}.$$

Then state the horizontal asymptotes of  $U$ , if any.

5. [10 pts.] Determine the intervals of continuity for the function  $F$  given by

$$F(x) = \begin{cases} x^3 + 4x + 1, & \text{if } x \leq 0 \\ 2x^3, & \text{if } x > 0 \end{cases}$$

6. Let  $f(x) = \frac{x-1}{x+2}$ .

- (a) [10 pts.] Use the limit definition of a derivative to find  $f'(1)$ .

- (b) [5 pts.] Determine an equation for the tangent line to the graph of  $f$  at the point  $(1, f(1))$ .

7. [10 pts.] Use the limit definition of a derivative to find  $v'(t)$  given that  $v(t) = \sqrt{2-4t}$ .