

1. 10 pts. Use the precise definition of limit to prove that

$$\lim_{x \rightarrow 7} (3x - 8) = 13.$$

2. 3 pts. each Let

$$f(x) = \begin{cases} 0, & x \leq -5 \\ \sqrt{25 - x^2}, & -5 < x < 5 \\ 3x, & x \geq 5. \end{cases}$$

Compute each limit, if it exists.

(a)  $\lim_{x \rightarrow -5} f(x)$       (b)  $\lim_{x \rightarrow 5^-} f(x)$       (c)  $\lim_{x \rightarrow 5^+} f(x)$       (d)  $\lim_{x \rightarrow 5} f(x)$       (e)  $\lim_{x \rightarrow 3} f(x)$

3. 10 pts. each Evaluate each limit algebraically using limit laws, showing work.

(a)  $\lim_{t \rightarrow 2} \frac{3t^2 - 7t + 2}{2 - t}$ .

(b)  $\lim_{x \rightarrow 49} \frac{\sqrt{x} - 7}{x - 49}$ .

4. 10 pts. Suppose

$$h(x) = \begin{cases} 3x + b, & x \leq 2 \\ x - 2, & x > 2. \end{cases}$$

Determine a value for  $b$  for which the limit  $\lim_{x \rightarrow 2} h(x)$  exists, and state the value of the limit.

5. 10 pts. Find all vertical asymptotes  $x = a$  of the function

$$f(x) = \frac{x^2 - 9x + 14}{x^2 - 5x + 6}.$$

For each value of  $a$  determine  $\lim_{x \rightarrow a^+} f(x)$ ,  $\lim_{x \rightarrow a^-} f(x)$ , and  $\lim_{x \rightarrow a} f(x)$ .

6. 10 pts. Evaluate the limit

$$\lim_{x \rightarrow \infty} \frac{4x^2 - 7}{8x^2 + 5x + 2}.$$

7. 15 pts. Determine  $\lim_{x \rightarrow \infty} f(x)$  and  $\lim_{x \rightarrow -\infty} f(x)$  for

$$f(x) = \frac{\sqrt{x^2 + 1}}{2x + 1}.$$

Then give the horizontal asymptotes of  $f$ , if any.

8. 10 pts. Show that  $f$  is not continuous at 4.

$$f(x) = \begin{cases} x^2 - 5, & \text{if } x \neq 4 \\ 13, & \text{if } x = 4 \end{cases}$$

9. 10 pts. Let  $g$  be given by

$$g(x) = \begin{cases} x^2 + x, & \text{if } x < 1 \\ a, & \text{if } x = 1 \\ 3x + 5, & \text{if } x > 1 \end{cases}$$

Find the value of  $a$  for which  $g$  is continuous from the left at 1, and the value of  $a$  for which  $g$  is continuous from the right at 1. Is there an  $a$  value for which  $g$  is continuous at 1?

10. Let  $f(x) = 3x^2 - 4x$ .

(a) 15 pts. Use the definition of derivative to find  $f'(1)$ .

(b) 5 pts. Determine an equation for the tangent line to the graph of  $f$  at the point  $(1, -1)$ .