# MATH 125 QUIZ #4 (SPRING 2021)

**1** Given $\csc\theta = 3$ and $\cot\theta < 0$, find the values of all trigonometric functions of $\theta$.

First, $r/y = \csc\theta = 3/1$ implies we can let $r = 3$ and $y = 1$. From $x^2 + y^2 = r^2$ we then obtain $x = \pm\sqrt{8}$. But $\cot\theta < 0$ implies (the terminal side of) $\theta$ is in QII or QIV (Q standing for quadrant), while $\csc\theta > 0$ implies $\theta$ is in QI or QII. Thus $\theta$ must be in QII, where $x < 0$, and so $x = -\sqrt{8} = -2\sqrt{2}$ must be the case. Finally we get

$$\sin\theta = \frac{1}{3}, \quad \cos\theta = -\frac{2\sqrt{2}}{3}, \quad \tan\theta = -\frac{\sqrt{2}}{4}, \quad \sec\theta = -\frac{3\sqrt{2}}{4}, \quad \cot\theta = -2\sqrt{2}.$$

**2** If $f(\theta) = \cos\theta$ and $f(a) = \frac{1}{4}$, what is $f(-a)$ and $f(a) + f(a+2\pi) + f(a-6\pi)$?

Since cos is an even function we have

$$f(-a) = \cos(-a) = \cos a = f(a) = \tfrac{1}{4},$$

and since cos is periodic with period $2\pi$ we have

$$f(a) + f(a+2\pi) + f(a-6\pi) = \cos a + \cos(a+2\pi) + \cos(a-6\pi) = 3\cos a = 3f(a) = \tfrac{3}{4}.$$

**3** Evaluate $\cos^{-1}\big(\cos\frac{15\pi}{7}\big)$ and $\tan(\tan^{-1} 10)$ exactly.

$\cos^{-1}\big(\cos\frac{15\pi}{7}\big) = \theta$ implies $\cos\theta = \cos\frac{15\pi}{7}$ for some $\theta \in [0,\pi]$. Now,

$$\cos\theta = \cos\tfrac{15\pi}{7} = \cos(\tfrac{\pi}{7} + 2\pi) = \cos\tfrac{\pi}{7},$$

and since $\frac{\pi}{7} \in [0,\pi]$, we conclude that $\cos^{-1}\big(\cos\frac{15\pi}{7}\big) = \theta = \frac{\pi}{7}$.

**4** Given $f(x) = \cos(x+2) + 1$, $-2 \le x \le \pi - 2$, find $f^{-1}$, and also find the domain and range of $f$ and $f^{-1}$.

Let $y = f(x)$, so that $y = \cos(x+2) + 1$, and then solving for $x$ gives

$$f^{-1}(y) = x = \cos^{-1}(y-1) - 2.$$

Also $\operatorname{Ran} f^{-1} = \operatorname{Dom} f = [-2, \pi - 2]$ and $\operatorname{Ran} f = \operatorname{Dom} f^{-1} = \{y : -1 \le y - 1 \le 1\} = [0, 2]$.