Math 125 Quiz #3 (Spring 2021)

1a Given $f(x) = \dfrac{3x-1}{2-x}$, find f^{-1}.

Letting $y = f(x)$, so that $y = \dfrac{3x-1}{2-x}$, solve for x to get $x = \dfrac{2y+1}{y+3}$, and therefore

$$f^{-1}(y) = \frac{2y+1}{y+3}.$$

1b Find the domain and range of f and f^{-1}.

$\operatorname{Ran} f^{-1} = \operatorname{Dom} f = (-\infty, 2) \cup (2, \infty)$ and $\operatorname{Ran} f = \operatorname{Dom} f^{-1} = (-\infty, -3) \cup (-3, \infty)$.

2 Find g^{-1} given that $g(x) = \dfrac{4x}{x^2+16}$, $x \geq 4$.

Let $y = g(x)$, so $y = \dfrac{4x}{x^2+16}$, and manipulate to get $yx^2 - 4x + 16y = 0$. We can solve this for x using the Quadratic Formula:

$$x = \frac{-(-4) \pm \sqrt{(-4)^2 - 4(y)(16y)}}{2y} = \frac{4 \pm \sqrt{16 - 64y^2}}{2y} = \frac{2 \pm 2\sqrt{1-4y^2}}{y}.$$

Given that $x \geq 4$ (that is, the domain of g is taken to be restricted to $[4, \infty)$), we must resolve the $\pm$ into $+$, and thereby obtain

$$g^{-1}(y) = \frac{2 + 2\sqrt{1-4y^2}}{y}.$$

3 Solve $3^{x^2+x} = \sqrt{3}$.

Write as $3^{x^2+x} = 3^{1/2}$, so that $x^2 + x = \frac{1}{2}$, and hence $2x^2 + 2x - 1 = 0$. Then with the Quadratic Formula (or completing the square) we get

$$x = \frac{-1 \pm \sqrt{3}}{2}.$$