Math 125 Quiz #2 (Spring 2021)

1 Form a polynomial function f having zeros -2 (multiplicity 1) and -1 (multiplicity 2), with degree 3 and such that $f(0) = 6$.

Start with $f(x) = c(x+2)(x+1)^2$. Now, $6 = f(0) = 2c$, so that $c = 3$ and we finally obtain $f(x) = 3(x+2)(x+1)^2$.

2 Use synthetic division to find the remainder when $f(x) = x^6 - 16x^4 + x^2 - 16$ is divided by $x + 4$. Then use relevant theorems to find whether $x + 4$ is a factor of $f(x)$. What is the other factor?

Divide $f(x)$ by $x + 4$:

$$\begin{array}{r|rrrrrr|r} -4 & 1 & 0 & -16 & 0 & 1 & 0 & -16 \\ & & -4 & 16 & 0 & 0 & -4 & 16 \\ \hline & 1 & -4 & 0 & 0 & 1 & -4 & 0 \end{array}$$

From this we see the remainder of the division is 0. By the Remainder Theorem $f(-4) = 0$, and so by the Factor Theorem $x + 4$ is a factor of $f(x)$. The other factor is $x^5 - 4x^4 + x - 4$.

3 Use the Rational Zeros Theorem to list the possible rational zeros of the polynomial function $f(x) = x^4 - x^3 - 6x^2 + 4x + 8$, then completely factor $f(x)$ and find all zeros of f.

Possible rational zeros are $\pm 1, \pm 2, \pm 4, \pm 8$. Testing -1, divide $f(x)$ by $x + 1$:

$$\begin{array}{r|rrrr|r} -1 & 1 & -1 & -6 & 4 & 8 \\ & & -1 & 2 & 4 & -8 \\ \hline & 1 & -2 & -4 & 8 & 0 \end{array}$$

Thus we find

$$\begin{aligned} f(x) &= (x+1)(x^3 - 2x^2 - 4x + 8) \\ &= (x+1)[x^2(x-2) - 4(x-2)] \\ &= (x+1)(x-2)(x^2 - 4) \\ &= (x+1)(x-2)^2(x+2). \end{aligned}$$

The zeros of f are -1, -2, and 2 (multiplicity 2).