Math 125 Quiz #2 (Fall 2020)

1 Form a degree 3 polynomial function with zeros -4, 0, 2.

We may let
$$f(x) = Cx(x+4)(x-2)$$
for any choice of constant $C \neq 0$.

2 Form a polynomial function with zeros -5, -1, 2, 6 and point $(\frac{5}{2}, 15)$ lying on its graph.

We start with
$$f(x) = C(x+5)(x+1)(x-2)(x-6),$$
with constant C chosen so that $f(\frac{5}{2}) = 15$; that is,
$$15 = f(\tfrac{5}{2}) = C(\tfrac{5}{2}+5)(\tfrac{5}{2}+1)(\tfrac{5}{2}-2)(\tfrac{5}{2}-6) = C(\tfrac{15}{2})(\tfrac{7}{2})(\tfrac{1}{2})(-\tfrac{7}{2}).$$
Spare yourself computational inconvenience by canceling 15 from both sides to get
$$C(\tfrac{1}{2})(\tfrac{7}{2})(\tfrac{1}{2})(-\tfrac{7}{2}) = 1,$$
and thus $C = -\frac{16}{49}$. Therefore
$$f(x) = -\tfrac{16}{49}(x+5)(x+1)(x-2)(x-6).$$

3 Solve $2x^3 - 3x^2 - 3x - 5 = 0$.

Let $f(x) = 2x^3 - 3x^2 - 3x - 5$. Possible rational zeros of f are $\pm 1, \pm 5, \pm\frac{5}{2}, \pm\frac{1}{2}$. With some trial-and-error we find that $\frac{5}{2}$ is a zero, and dividing $f(x) \div (x - \frac{5}{2})$ gives the quotient $2x^2 + 2x + 2$. Thus
$$f(x) = (x - \tfrac{5}{2})(2x^2 + 2x + 2) = (2x-5)(x^2+x+1).$$
The equation to solve, $f(x) = 0$, now becomes
$$(2x-5)(x^2+x+1) = 0,$$
satisfied if either $2x - 5 = 0$ or $x^2 + x + 1 = 0$. The former equation gives $x = \frac{5}{2}$ of course, while the latter can be solved with the quadratic formula to give $x = -\frac{1}{2} \pm \frac{\sqrt{3}}{2}i$. Solution set is
$$\left\{ \frac{5}{2}, -\frac{1}{2} \pm \frac{\sqrt{3}}{2}i \right\}.$$