MATH 125
SUMMER 2018 NAME:
EXAM 4

1. [10 pts.] Find the exact value of $\cos^{-1}\left[\cos\left(-\frac{5\pi}{3}\right)\right]$ and $\csc\left[\tan^{-1}(-2)\right]$.

2. [10 pts.] Find the inverse function f^{-1} of

$$f(x) = 2\tan x - 3; \quad -\frac{\pi}{2} < x < \frac{\pi}{2}.$$

Also find the range of f, and the domain and range of f^{-1}.

3. [10 pts. each] Solve each trigonometric equation on the interval $0 \le \theta < 2\pi$.
   (a) $4\cos^2\theta - 3 = 0$
   (b) $\sqrt{3}\cot\theta + 1 = 0$
   (c) $(\cot\theta + 1)\left(\csc\theta - \frac{1}{2}\right) = 0$
   (d) $\tan\theta = 2\sin\theta$

4. [10 pts. each] Establish the identity.
   (a) $\tan^3 x + \tan x = \sec^2 x \tan x$
   (b) $1 - \dfrac{\sin^2 u}{1+\cos u} = \cos u.$

5. [10 pts.] Find the exact value using a sum or difference formula:

$$\sin\frac{\pi}{18}\cos\frac{5\pi}{18} + \cos\frac{\pi}{18}\sin\frac{5\pi}{18}$$

6. [10 pts.] Find the exact value of $\sin 195^\circ$ using a double-angle or half-angle formula.

7. [10 pts.] The hypotenuse of a right triangle is 3 meters. If one leg is one meter, find the degree measure of each angle.

8. [10 pts.] The angle of elevation of the sun is 35.1° at the instant the shadow cast by the Washington Monument is 789 feet long. Use this information to find the height of the monument to the nearest tenth of a foot.

9. [10 pts. each] Solve the triangle, rounding to the tenths place.
   (a) $C = 46^\circ 32'$, $a = 56.2$ m, $c = 22.1$ m.
   (b) $A = 10^\circ$, $a = 3$, $b = 10$.
   (c) $a = 4$, $b = 3$, $c = 6$.

TRIGONOMETRIC IDENTITIES

$$\sin(u \pm v) = \sin u \cos v \pm \cos u \sin v$$

$$\cos(u \pm v) = \cos u \cos v \mp \sin u \sin v$$

$$\tan(u \pm v) = \frac{\tan u \pm \tan v}{1 \mp \tan u \tan v}$$

$$\sin 2x = 2 \sin x \cos x$$

$$\cos 2x = \cos^2 x - \sin^2 x$$

$$\tan 2x = \frac{2 \tan x}{1 - \tan^2 x}$$

$$\sin \frac{x}{2} = \pm \sqrt{\frac{1 - \cos x}{2}}$$

$$\cos \frac{x}{2} = \pm \sqrt{\frac{1 + \cos x}{2}}$$

$$\tan \frac{x}{2} = \frac{\sin x}{1 + \cos x}$$

$$\sin x \sin y = \tfrac{1}{2}[\cos(x - y) - \cos(x + y)]$$

$$\cos x \cos y = \tfrac{1}{2}[\cos(x - y) + \cos(x + y)]$$

$$\sin x \cos y = \tfrac{1}{2}[\sin(x - y) + \sin(x + y)]$$

$$\cos x \sin y = \tfrac{1}{2}[\sin(x + y) - \sin(x - y)]$$

$$\sin x + \sin y = 2 \sin \frac{x + y}{2} \cos \frac{x - y}{2}$$

$$\sin x - \sin y = 2 \cos \frac{x + y}{2} \sin \frac{x - y}{2}$$

$$\cos y + \cos x = 2 \cos \frac{x + y}{2} \cos \frac{x - y}{2}$$

$$\cos y - \cos x = 2 \sin \frac{x + y}{2} \sin \frac{x - y}{2}$$