

MATH 125
SPRING 2022
EXAM 4

NAME:

1. 10 pts. Given that $\csc \theta = 3$ and $\cot \theta < 0$, find the value of all trigonometric functions of θ .
2. 10 pts. each Find the exact value of each expression, or state that it is undefined.

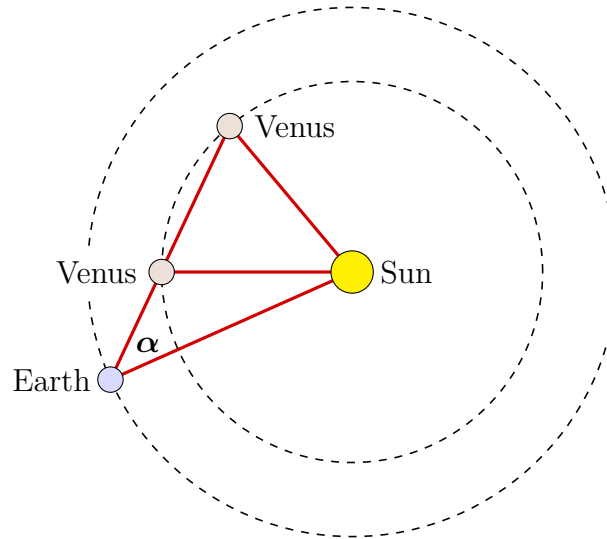
 - (a) $\cos^{-1}\left(\cos \frac{3\pi}{10}\right)$
 - (b) $\sin^{-1}\left(\sin \frac{6\pi}{7}\right)$
 - (c) $\csc\left(\tan^{-1} \frac{1}{2}\right)$
 - (d) $\sec^{-1}\left(\sin \frac{7\pi}{6}\right)$
3. 15 pts. Find the inverse function f^{-1} for $f(x) = -2\cos(3x)$, $0 \leq x \leq \pi/3$. Using interval notation, find the range of f and the domain and range of f^{-1} .
4. 10 pts. each Solve each trigonometric equation on the interval $0 \leq \theta < 2\pi$.

 - (a) $3 \tan^3 \theta = \tan \theta$
 - (b) $\sec^2 \theta - 2 = 0$
 - (c) $4(1 + \sin \theta) = \cos^2 \theta$
5. 10 pts. each Establish the identity.

 - (a) $\frac{\csc u - \cot u}{\sec u - 1} = \cot u$
 - (b) $\frac{1}{1 - \sin \theta} = \sec^2 \theta + \tan \theta \sec \theta$
6. 10 pts. A laser beam is to be aimed toward the center of the moon, but the beam strays 0.4° from its intended path. Given that the moon is 384,400 km from Earth, by how much will the beam miss its target? Round to the nearest hundred kilometers.
7. 10 pts. From the top of a 70-meter-tall lighthouse, the angle of depression to a ship in the ocean is 21° . How far is the ship from the base of the lighthouse? Round to the nearest meter.
8. 10 pts. each Solve the triangle using the Law of Sines, rounding sides to the hundredths place and angles to the tenths place when necessary. If no triangle results from the given information, then explain why.

 - (a) $A = 50^\circ$, $C = 20^\circ$, $a = 3$.
 - (b) $a = 2$, $c = 1$, $C = 100^\circ$.

9. 15 pts. The distance from the Sun to Earth is about 149,600,000 km, and the distance from the Sun to Venus is about 108,200,000 km. The elongation angle α is the angle formed between the line of sight from Earth to the Sun and the line of sight from Earth to Venus. Assuming that $\alpha = 40^\circ$, find the possible distances between Earth and Venus. Round to the nearest 100,000 km.



TRIGONOMETRIC IDENTITIES

$$\sin(u \pm v) = \sin u \cos v \pm \cos u \sin v$$

$$\cos(u \pm v) = \cos u \cos v \mp \sin u \sin v$$

$$\tan(u \pm v) = \frac{\tan u \pm \tan v}{1 \mp \tan u \tan v}$$

$$\sin 2u = 2 \sin u \cos u$$

$$\cos 2u = \cos^2 u - \sin^2 u$$

$$\tan 2u = \frac{2 \tan u}{1 - \tan^2 u}$$

$$\sin \frac{u}{2} = \pm \sqrt{\frac{1 - \cos u}{2}}$$

$$\cos \frac{u}{2} = \pm \sqrt{\frac{1 + \cos u}{2}}$$

$$\tan \frac{u}{2} = \frac{\sin u}{1 + \cos u}$$