MATH 125
SPRING 2013 NAME:
EXAM 4

1. [5 pts. each] Carry out the conversion, showing work.
   (a) 12.5° to radians. Leave answer in terms of π.
   (b) 1 to degrees. Round answer to two decimal places.

2. [10 pts.] Determine the amplitude, period, and phase shift of the function $y = 4\sin\left(\frac{1}{4}x + \frac{\pi}{8}\right)$

3. [10 pts.] Simplify: $\dfrac{5\cos\varphi}{\sin^2\varphi} \cdot \dfrac{\sin^2\varphi - \sin\varphi\cos\varphi}{\sin^2\varphi - \cos^2\varphi}$

4. [10 pts.] Use a half-angle identity to evaluate $\cos 15^\circ$ exactly.

5. [10 pts. each] Establish each identity.
   (a) $\dfrac{1+\cos^2 x}{\sin^2 x} = 2\csc^2 x - 1$
   (b) $\dfrac{1+\sin x}{1-\sin x} = (\sec x + \tan x)^2$

6. [10 pts. each] Find the exact value of each.
   (a) $\sin\left(\tan^{-1}\frac{\sqrt{3}}{3}\right)$
   (b) $\sin^{-1}\left(\sin\frac{7\pi}{6}\right)$

7. [10 pts. each] Solve, finding all solutions in $[0, 2\pi)$.
   (a) $2\sin^2\theta + 7\sin\theta = 4$
   (b) $\sin 2x\cos x - \sin x = 0$
   (c) $\sec^2 t - 2\tan^2 t = 0$

8. [10 pts. each] Solve the triangle.
   (a) $B = 10^\circ$, $C = 100^\circ$, $b = 2$.
   (b) $A = 89^\circ$, $a = 15.6$, $b = 18.4$.
   (c) $A = 10^\circ$, $a = 3$, $b = 10$.
   (d) $a = 4$, $b = 3$, $c = 6$.

TRIGONOMETRIC IDENTITIES

$$\sin(u \pm v) = \sin u \cos v \pm \cos u \sin v$$

$$\cos(u \pm v) = \cos u \cos v \mp \sin u \sin v$$

$$\tan(u \pm v) = \frac{\tan u \pm \tan v}{1 \mp \tan u \tan v}$$

$$\sin 2x = 2 \sin x \cos x$$

$$\cos 2x = \cos^2 x - \sin^2 x$$

$$\tan 2x = \frac{2 \tan x}{1 - \tan^2 x}$$

$$\sin \frac{x}{2} = \pm \sqrt{\frac{1 - \cos x}{2}}$$

$$\cos \frac{x}{2} = \pm \sqrt{\frac{1 + \cos x}{2}}$$

$$\tan \frac{x}{2} = \frac{\sin x}{1 + \cos x}$$

$$\sin x \sin y = \tfrac{1}{2}[\cos(x - y) - \cos(x + y)]$$

$$\cos x \cos y = \tfrac{1}{2}[\cos(x - y) + \cos(x + y)]$$

$$\sin x \cos y = \tfrac{1}{2}[\sin(x - y) + \sin(x + y)]$$

$$\cos x \sin y = \tfrac{1}{2}[\sin(x + y) - \sin(x - y)]$$

$$\sin x + \sin y = 2 \sin \frac{x + y}{2} \cos \frac{x - y}{2}$$

$$\sin x - \sin y = 2 \cos \frac{x + y}{2} \sin \frac{x - y}{2}$$

$$\cos y + \cos x = 2 \cos \frac{x + y}{2} \cos \frac{x - y}{2}$$

$$\cos y - \cos x = 2 \sin \frac{x + y}{2} \sin \frac{x - y}{2}$$