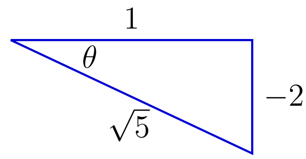
Math 125 Exam #4 Key (Summer 2018)

1 Find $\theta \in [0, \pi]$ such that $\theta = \cos^{-1}\left[\cos\left(-\frac{5\pi}{3}\right)\right]$, or equivalently $\cos\theta = \cos\left(-\frac{5\pi}{3}\right)$. The only answer is $\theta = \frac{\pi}{3}$.

Next, let $\theta = \tan^{-1}(-2)$, so $\theta \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$ is such that $\tan\theta = -2$. Thus we have a triangle in Quadrant IV:



From this we can see that $\csc\left[\tan^{-1}(-2)\right] = \csc\theta = -\frac{\sqrt{5}}{2}$.

2 The inverse function is

$$f^{-1}(x) = \tan^{-1}\left(\frac{x+3}{2}\right).$$

Range of f: $(-\infty, \infty)$. Domain of f^{-1}: $(-\infty, \infty)$. Range of f^{-1}: $(-\frac{\pi}{2}, \frac{\pi}{2})$.

3a From $\cos^2\theta = \frac{3}{4}$ we get $\cos\theta = \pm\frac{\sqrt{3}}{2}$. Solutions: $\theta = \frac{\pi}{6}, \frac{5\pi}{6}, \frac{7\pi}{6}, \frac{11\pi}{6}$.

3b From $\cot\theta = -\frac{1}{\sqrt{3}}$ we get $\theta = \frac{2\pi}{3}, \frac{5\pi}{3}$.

3c Either $\cot\theta = -1$ or $\csc\theta = \frac{1}{2}$. The latter implies that $\sin\theta = 2$, which has no solution. Thus the only solutions are: $\theta = \frac{3\pi}{4}, \frac{7\pi}{4}$.

3d We have

$$\tan\theta = 2\sin\theta \quad \Rightarrow \quad \frac{\sin\theta}{\cos\theta} - 2\sin\theta = 0 \quad \Rightarrow \quad (\sin\theta)(\sec\theta - 2) = 0.$$

Solutions: $\theta = 0, \pi, \frac{\pi}{3}, \frac{5\pi}{3}$.

4a We have

$$\tan^3 x + \tan x = (\tan x)(\tan^2 x + 1) = (\tan x)(\sec^2 x) = \sec^2 x \tan x.$$

4b We have

$$\begin{aligned} 1 - \frac{\sin^2 u}{1+\cos u} &= \frac{(1+\cos u) - \sin^2 u}{1+\cos u} = \frac{(1+\cos u) - (1-\cos^2 u)}{1+\cos u} \\ &= \frac{\cos^2 u + \cos u}{1+\cos u} = \frac{(\cos u)(\cos u + 1)}{1+\cos u} = \cos u. \end{aligned}$$

5 With the identity $\sin(u+v) = \sin u \cos v + \cos u \sin v$,

$$\sin\frac{\pi}{18}\cos\frac{5\pi}{18} + \cos\frac{\pi}{18}\sin\frac{5\pi}{18} = \sin\left(\frac{\pi}{18}+\frac{5\pi}{18}\right) = \sin\frac{\pi}{3} = \frac{\sqrt{3}}{2}.$$

6 With a half-angle identity,

$$\sin 195^\circ = -\sqrt{\frac{1-\cos 390^\circ}{2}} = -\sqrt{\frac{1-\cos 30^\circ}{2}} = -\sqrt{\frac{1-\frac{\sqrt{3}}{2}}{2}} = -\frac{\sqrt{2-\sqrt{3}}}{2}.$$

7 Angles are 90°, $\sin^{-1}(1/3) \approx 19.47^\circ$, and $90^\circ - \sin^{-1}(1/3) \approx 70.53^\circ$.

8 Let h be the height of the monument. Then

$$\tan 35.1^\circ = \frac{h}{789} \quad\Rightarrow\quad h = 789\tan 35.1^\circ \approx 554.5 \text{ ft}$$

9a By the Law of Sines:

$$\sin A = \frac{a\sin C}{c} = \frac{56.2\sin(46^\circ 32')}{22.1} \approx 1.85.$$

There is no solution.

9b We have

$$\begin{aligned}\frac{\sin B}{10} = \frac{\sin 10^\circ}{3} \quad&\Rightarrow\quad \sin B = 0.57883\\ &\Rightarrow\quad \sin^{-1}(\sin B) = \sin^{-1}(0.57883) = 35.366^\circ\\ &\Rightarrow\quad \sin B = \sin 35.366^\circ.\end{aligned}$$

One solution to this equation is of course $B_1 = 35.366^\circ$; however B could also be the Quadrant II angle

$$B_2 = 180^\circ - \sin^{-1}(0.57883) = 144.63^\circ$$

(see very pretty picture below).

For the angle B_1 we get $C_1 = 134.634^\circ$, and then by the Law of Cosines we obtain

$$c_1^2 = a^2 + b^2 - 2ab\cos C_1 = 3^2 + 10^2 - 2(3)(10)\cos 134.634^\circ = 151.156 \quad\Rightarrow\quad c_1 = 12.29.$$

So one possible triangle (rounding to the nearest tenth) has

$$B_1 = 35.4^\circ, \quad C_1 = 134.6^\circ, \quad c_1 = 12.3.$$

For the angle B_2 we get $C_2 = 25.370^\circ$, and then by the Law of Cosines we obtain

$$c_2^2 = a^2 + b^2 - 2ab\cos C_2 = 3^2 + 10^2 - 2(3)(10)\cos 25.370^\circ = 54.786 \quad\Rightarrow\quad c_1 = 7.40.$$

So another possible triangle has

$$B_2 = 144.6^\circ, \quad C_2 = 25.4^\circ, \quad c_2 = 7.4.$$

9c The Law of Cosines is necessary here:

$$\begin{aligned} c^2 = a^2 + b^2 - 2ab\cos C \quad &\Rightarrow \quad 6^2 = 4^2 + 3^2 - 2(4)(3)\cos C \\ &\Rightarrow \quad \cos C = -11/24 \\ &\Rightarrow \quad C = \cos^{-1}(-11/24) \approx 117.28^\circ. \end{aligned}$$

And

$$\begin{aligned} b^2 = a^2 + c^2 - 2ac\cos B \quad &\Rightarrow \quad 3^2 = 4^2 + 6^2 - 2(4)(6)\cos B \\ &\Rightarrow \quad \cos B = 43/48 \\ &\Rightarrow \quad B = \cos^{-1}(43/48) \approx 26.38^\circ. \end{aligned}$$

Finally, $A = 180^\circ - 26.38^\circ - 117.28^\circ = 36.34^\circ$. To the nearest tenth we have:

$$A = 36.3^\circ, \quad B = 26.4^\circ, \quad C = 117.3^\circ.$$