

MATH 125 EXAM #3 KEY (SUMMER 2018)

**1** Domain is  $(-\infty, \infty)$  and range is  $(2, \infty)$ .

**2** Equation becomes  $e^{x^2+4x} = e^{12}$ , and so  $x^2 + 4x = 12$ . This solves to give solution set  $\{-6, 2\}$ .

**3** Domain is

$$\left\{x : \frac{1}{x+2} > 0\right\} = \{x : x > -2\} = (-2, \infty).$$

**4** We have  $e^{x/5} = 9/5$ , giving  $x/5 = \ln(9/5)$ , and hence  $x = 5 \ln(9/5)$ .

**5a** Find  $h$  such that  $320 = 760e^{-0.145h}$ , which implies  $-0.145h = \ln(320/760)$ , and hence

$$h = -\frac{1}{0.145} \ln\left(\frac{8}{19}\right) \approx 5.97 \text{ km.}$$

**5b** Find  $h$  such that  $200 = 760e^{-0.145h}$ , which implies  $-0.145h = \ln(200/760)$ , and hence

$$h = -\frac{1}{0.145} \ln\left(\frac{5}{19}\right) \approx 9.21 \text{ km.}$$

**6** With laws of logarithms:

$$\log_2 \frac{(x-3)^3}{(2x-1)(x+1)}$$

**7a** We have

$$\log_6(x+4)(x+3) = 1 \Rightarrow (x+4)(x+3) = 6 \Rightarrow x = -6, -1.$$

The value  $-6$  is an extraneous solution, and so the solution set is  $\{-1\}$ .

**7b** Taking logarithms of both sides:

$$x \ln(3/5) = (1-x) \ln 7 \Rightarrow x = \frac{\ln 7}{\ln(3/5) + \ln 7} = \frac{\ln 7}{\ln(21/5)}.$$

**8a**  $A(11) = 100e^{-0.087(11)} \approx 38.4 \text{ g.}$

**8b** Find  $t$  for which  $A(t) = \frac{1}{2}A_0$ :

$$\frac{1}{2}A_0 = A_0e^{-0.087t} \Rightarrow e^{-0.087t} = \frac{1}{2} \Rightarrow -0.087t = \ln(1/2) \Rightarrow t \approx 7.97 \text{ days.}$$

Note that the value of  $A_0$  is irrelevant.

**9**  $(210^\circ)(\pi/180^\circ) = 7\pi/6$ .

**10** Circumference of circle is  $4\pi$ , so the object travels  $\frac{5}{4\pi}$  of the circle in 20 seconds, and hence sweeps out  $\theta = (2\pi)\left(\frac{5}{4\pi}\right) = \frac{5}{2}$  radians in 20 seconds. Angular speed is therefore

$$\omega = \frac{\theta}{t} = \frac{5/2}{20} = \frac{1}{8} \text{ radians/second.}$$

Linear speed:  $v = s/t = \frac{5}{20} \text{ m/s} = \frac{1}{4} \text{ m/s}$ .

**11**  $\sin \theta = -\frac{12}{13}$ ,  $\cos \theta = \frac{5}{13}$ ,  $\tan \theta = -\frac{12}{5}$ ,  $\csc \theta = -\frac{13}{12}$ ,  $\sec \theta = \frac{13}{5}$ ,  $\cot \theta = -\frac{5}{12}$ .

**12**  $\cos \theta = -\frac{\sqrt{5}}{3}$ ,  $\tan \theta = \frac{2}{\sqrt{5}}$ ,  $\csc \theta = -\frac{3}{2}$ ,  $\sec \theta = -\frac{3}{\sqrt{5}}$ ,  $\cot \theta = \frac{\sqrt{5}}{2}$ .

**13** Domain is  $(-\infty, \infty)$ , and range is  $[-3, 5]$ .

**14** Domain is

$$\left\{x : \frac{3\pi}{2}x \neq \frac{\pi}{2} + k\pi\right\} = \left\{x : x \neq \frac{2k+1}{3}\right\},$$

where  $k$  is any integer. Range  $(-\infty, -3] \cup [3, \infty)$ .

**15**  $y = 2 \sin(x/2)$ .

**16**  $y = 5 \sin(4x - 8)$ .