

MATH 125 EXAM #2 KEY (SUMMER 2018)

1 In general

$$(x + y)^n = \sum_{k=0}^n \binom{n}{k} x^{n-k} y^k,$$

so

$$\begin{aligned} (2x - 1)^6 &= \sum_{k=0}^6 \binom{6}{k} (2x)^{6-k} (-1)^k \\ &= (2x)^6 - 6(2x)^5 + 15(2x)^4 - 20(2x)^3 + 15(2x)^2 - 6(2x) + 1 \\ &= 64x^6 - 192x^5 + 240x^4 - 160x^3 + 60x^2 - 12x + 1. \end{aligned}$$

2 $f(x) = x(x + 3)(x - 6)$.

3a $\frac{5}{2}$ (multiplicity 4), -3 (multiplicity 9).

3b x -intercepts are $\frac{5}{2}$ and -3 , and the graph of f touches the x -axis at $\frac{5}{2}$ and crosses at -3 .

3c There are at most 12 turning points, since the polynomial is of degree 13.

3d $f(x) \rightarrow +\infty$ as $x \rightarrow +\infty$, and $f(x) \rightarrow -\infty$ as $x \rightarrow -\infty$.

4 Possible rational zeros are $\pm 1, \pm 2, \pm \frac{1}{2}$. Synthetic division shows 1 to be a zero:

$$\begin{array}{r|rrrrr} 1 & 2 & -1 & -5 & 2 & 2 \\ & & 2 & 1 & -4 & -2 \\ \hline & 2 & 1 & -4 & -2 & 0 \end{array} \longrightarrow 2x^3 + x^2 - 4x - 2.$$

So $f(x) = (x - 1)(2x^3 + x^2 - 4x - 2)$. For $g(x) = 2x^3 + x^2 - 4x - 2$ synthetic division shows $-\frac{1}{2}$ to be a zero:

$$\begin{array}{r|rrrrr} -\frac{1}{2} & 2 & 1 & -4 & -2 \\ & & -1 & 0 & 2 \\ \hline & 2 & 0 & -4 & 0 \end{array} \longrightarrow 2x^2 - 4.$$

So

$$\begin{aligned} f(x) &= (x - 1)\left(x + \frac{1}{2}\right)(2x^2 - 4) = (x - 1)\left(x + \frac{1}{2}\right) \cdot 2(x^2 - 2) \\ &= (x - 1)(2x + 1)(x - \sqrt{2})(x + \sqrt{2}), \end{aligned}$$

and the zeros of f are $1, -\frac{1}{2}, -\sqrt{2}, \sqrt{2}$.

5 Multiply by 2: $2x^3 + 3x^2 + 6x - 4 = 0$. Possible rational zeros of the polynomial are $\pm 1, \pm 2, \pm 4, \pm \frac{1}{2}$. Synthetic division shows $\frac{1}{2}$ to be a zero:

$$\begin{array}{r|rrrr} \frac{1}{2} & 2 & 3 & 6 & -4 \\ & & 1 & 2 & 4 \\ \hline & 2 & 4 & 8 & 0 \end{array}$$

Thus the polynomial factors as $(x - \frac{1}{2})(2x^2 + 4x + 8)$. Equation becomes:

$$(x - \frac{1}{2})(2x^2 + 4x + 8) = 0.$$

Since

$$2x^2 + 4x + 8 = 2(x^2 + 2x + 4) = 2[(x^2 + 2x + 1) + 3] = 2[(x + 1)^2 + 3],$$

$2x^2 + 4x + 8$ has no real zeros, and so $\frac{1}{2}$ is the only real-valued solution to the original equation.

6 Another zero must be $3 + 2i$ since the polynomial functions has real coefficients. This means that

$$[x - (3 - 2i)][x - (3 + 2i)] = x^2 - 6x + 13$$

is a factor of $f(x)$. With long division we have

$$\frac{f(x)}{x^2 - 6x + 13} = x^2 - 3x - 10,$$

and so $f(x) = (x^2 - 6x + 13)(x^2 - 3x - 10)$. The other zeros of f must be the zeros of $x^2 - 3x - 10$. Since $x^2 - 3x - 10 = (x - 5)(x + 2)$ has zeros $-2, 5$, the zeros of f are $3 - 2i, 3 + 2i, -2, 5$.

7a Domain of Q is

$$\{x : x^2 + x - 6 \neq 0\} = \{x : x \neq -3, 2\} = (-\infty, -3) \cup (-3, 2) \cup (2, \infty).$$

7b The only intercept of Q is $(0, 0)$.

7c $x = -3$ and $x = 2$.

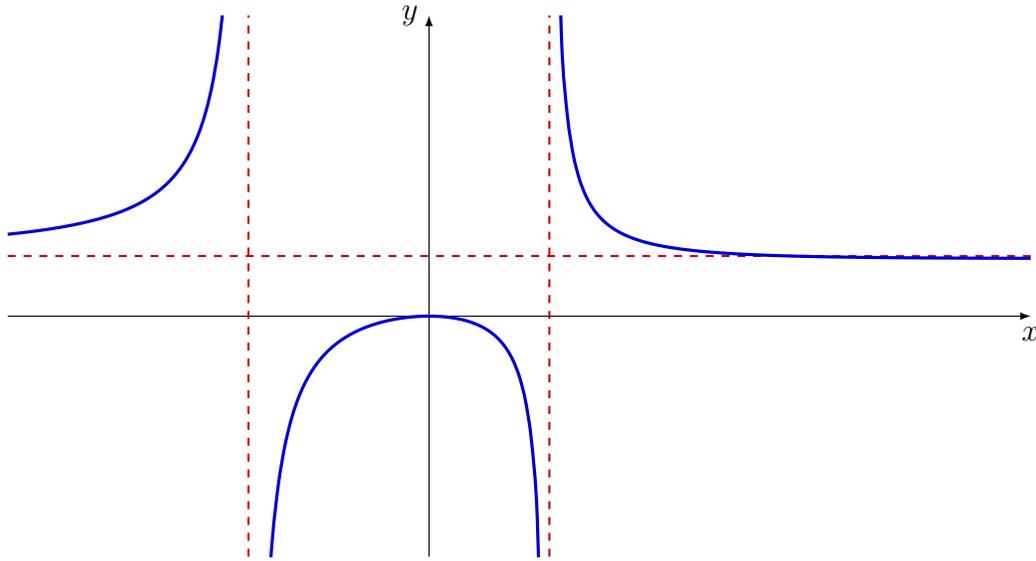
7d $y = 1$ is the horizontal asymptote. No oblique asymptote.

7e Find any x for which $Q(x) = 1$:

$$\frac{x^2}{x^2 + x - 6} = 1 \Rightarrow x^2 = x^2 + x - 6 \Rightarrow x = 6.$$

Thus the graph of Q crosses the horizontal asymptote at $(6, 1)$.

7f Graph:



8a We have

$$3x^3 + 15x^2 < 0 \Rightarrow 3x^2(x + 5) < 0 \Rightarrow x + 5 < 0 \Rightarrow x < -5,$$

so $x \in (-\infty, -5)$.

8b For $x \neq 2$ we may divide by $(x - 2)^2$ to obtain:

$$\frac{1}{x^2 - 1} > 0 \Rightarrow x^2 - 1 > 0 \Rightarrow x^2 > 1 \Rightarrow |x| > 1,$$

and so either $x > 1$ or $x < -1$. Noting that $x = 2$ satisfies the inequality as well, the solution set is $(-\infty, -1) \cup (1, \infty)$.

9 The domain of f consists of all x for which $(x-2)/(2x+5) \geq 0$. We have $(x-2)/(2x+5) \geq 0$ if $x-2 \geq 0$ and $2x+5 > 0$, which occurs if and only if $x \geq 2$. We also have $(x-2)/(2x+5) \geq 0$ if $x-2 \leq 0$ and $2x+5 < 0$, which occurs if and only if $x < -\frac{5}{2}$. Thus

$$\text{Dom } f = \left\{ x : \frac{x-2}{2x+5} \geq 0 \right\} = (-\infty, -\frac{5}{2}) \cup [2, \infty).$$

10a We have $(f \circ g)(x) = f(g(x)) = f(1 - 7x) = \sqrt{(1 - 7x) + 9} = \sqrt{10 - 7x}$. Domain:

$$\begin{aligned} \text{Dom } f \circ g &= \{x : x \in \text{Dom } g \ \& \ g(x) \in \text{Dom } f\} \\ &= \{x : x \in \mathbb{R} \ \& \ 1 - 7x \in [-9, \infty)\} \\ &= \{x : 1 - 7x \geq -9\} = (-\infty, \frac{10}{7}]. \end{aligned}$$

10b We have $(f \circ f)(x) = f(f(x)) = f(\sqrt{x+9}) = \sqrt{\sqrt{x+9}+9}$. Domain:

$$\begin{aligned} \text{Dom } f \circ f &= \{x : x \in \text{Dom } f \ \& \ f(x) \in \text{Dom } f\} \\ &= \{x : x \in [-9, \infty) \ \& \ \sqrt{x+9} \in [-9, \infty)\} \\ &= [-9, \infty). \end{aligned}$$

11a Solve $y = x^2 - 36$ for x , noting that $x \leq 0$ is given:

$$x^2 = y + 36 \Rightarrow |x| = \sqrt{y + 36} \Rightarrow -x = \sqrt{y + 36} \Rightarrow x = -\sqrt{y + 36},$$

and so $f^{-1}(y) = -\sqrt{y + 36}$. We can also write this as $f^{-1}(x) = -\sqrt{x + 36}$.

11b Solve $y = (4x - 1)/(3x + 8)$ for x to get

$$x = \frac{8y + 1}{4 - 3y}.$$

Thus

$$f^{-1}(x) = \frac{8x + 1}{4 - 3x}.$$