Math 125 Exam #1 Key (Summer 2018)

1 $f(-3) = \sqrt{(-3)^2 - (-3)} = \sqrt{12} = 2\sqrt{3}$ and $f(-2x) = \sqrt{(-2x)^2 - (-2x)} = \sqrt{4x^2 + 2x}$.

2a Domain of g: $(-\infty, \frac{3}{7}) \cup (\frac{3}{7}, \infty)$.

2b Domain of h: $[4, 7) \cup (7, \infty)$.

3a We have

$$(f+g)(x) = \left(1 + \frac{1}{x}\right) + \frac{1}{x+3} = \frac{x^2 + 5x + 3}{x(x+3)},$$

with domain $\{x : x \neq -3, 0\}$.

3b We have

$$(f/g)(x) = \frac{1 + \frac{1}{x}}{\frac{1}{x+3}} = x + 3 + \frac{x+3}{x} = x + 4 + \frac{3}{x},$$

with domain $\{x : x \neq -3, 0\}$.

4a Since $p(-1) = -3(-1)^2 + 5(-1) = -8 \neq 2$, the point $(-1, 2)$ is not on the graph of p.

4b $p(x) = -2$ implies $-3x^2 + 5x = -2$, which solves to give $x = -\frac{1}{3}, 2$.

4c Domain of p is $(-\infty, \infty)$.

4d Since $p(0) = 0$, the y-intercept of p is $(0, 0)$.

4e We solve $p(x) = 0$, which yields $x = 0, \frac{5}{3}$.

5 Definition of f:

$$f(x) = \begin{cases} x, & x \in [-3, 0] \\ -\frac{3}{4}x + 4, & x \in (0, 4) \end{cases}$$

6a The rectangle's area is the sum of the areas of eight right triangles with legs of length x and $y = \sqrt{4 - x^2}$, and hypotenuse of length 2.

$$A(x) = 8\left(\tfrac{1}{2}x\sqrt{4 - x^2}\right) = 4x\sqrt{4 - x^2}.$$

6b Perimeter is $p(x) = 4x + 4y = 4x + 4\sqrt{4 - x^2}$.

7 Find the solutions to $f(x) = g(x)$, which becomes $(x + 6)(x - 4) = 0$, and hence $x = -6, 4$. Points of intersection are thus $(-6, f(-6)) = (-6, 3)$ and $(4, f(4)) = (4, 33)$.

8 The zeros of H are

$$H(x) = 0 \quad \Rightarrow \quad 3x^2 + x - \frac{1}{2} = 0 \quad \Rightarrow \quad \left(x + \frac{1}{6}\right)^2 = \frac{7}{36} \quad \Rightarrow \quad x = -\frac{1}{6} \pm \frac{\sqrt{7}}{6},$$

and so the x-intercepts are $\left(-\frac{1}{6} \pm \frac{\sqrt{7}}{6}, 0\right)$.

9 We have

$$x^2 + 5x + 6 \geq 0 \quad \Rightarrow \quad (x+2)(x+3) \geq 0,$$

implying either $x \geq -2$ or $x \leq -3$, and so the solution set is $(-\infty, -3) \cup (-2, \infty)$.

10 The only fitting quadratic function is

$$f(x) = \frac{75}{200^2}(x - 200)(x + 200) + 75.$$

Thus the height of the cables at the points 100 meters to either side of the center is

$$f(100) = 75\left[\frac{(-100)(300)}{200^2} + 1\right] = 18.75 \text{ m}.$$

11 Write $|4 - 5t| = 19$, so either $4 - 5t = 19$ or $4 - 5t = -19$. Solution set: $\{-3, \frac{23}{5}\}$.

12 Write $|y - 1| > 7$, so either $y - 1 < -7$ or $y - 1 > 7$. Solution set: $(-\infty, -6) \cup (8, \infty)$.