Math 125 Exam #2 Key (Summer 2017)

1 In general

$$(x+y)^n = \sum_{k=0}^{n} \binom{n}{k} x^{n-k} y^k,$$

so

$$\begin{aligned}(2x+3)^5 &= \sum_{k=0}^{5} \binom{5}{k} (2x)^{5-k}(3)^k \\ &= (2x)^5 + 5(2x)^4(3) + 10(2x)^3(3)^2 + 10(2x)^2(3)^3 + 5(2x)(3)^4 + 3^5 \\ &= 32x^5 + 240x^4 + 720x^3 + 1080x^2 + 810x + 243.\end{aligned}$$

2 $f(x) = (x+4)(x+1)(x-2)(x-3)$.

3a -4 (multiplicity 1), -3 (multiplicity 3).

3b x-intercepts are -4 and -3, and the graph of f crosses the x-axis at both.

3c There are at most 3 turning points.

3d $f(x) \to +\infty$ as $x \to \pm\infty$.

4 Possible rational zeros are $\pm 1, \pm 2, \pm\frac{1}{2}$. Synthetic division shows 1 to be a zero:

$$\begin{array}{c|rrrr|r} 1 & 2 & -1 & -5 & 2 & 2 \\ & & 2 & 1 & -4 & -2 \\ \hline & 2 & 1 & -4 & -2 & 0 \end{array} \quad \longrightarrow \quad 2x^3 + x^2 - 4x - 2.$$

So $f(x) = (x-1)(2x^3 + x^2 - 4x - 2)$. For $g(x) = 2x^3 + x^2 - 4x - 2$ synthetic division shows $-\frac{1}{2}$ to be a zero:

$$\begin{array}{c|rrr|r} -\frac{1}{2} & 2 & 1 & -4 & -2 \\ & & -1 & 0 & 2 \\ \hline & 2 & 0 & -4 & 0 \end{array} \quad \longrightarrow \quad 2x^2 - 4.$$

So

$$\begin{aligned} f(x) &= (x-1)\big(x + \tfrac{1}{2}\big)(2x^2 - 4) = (x-1)\Big(x + \frac{1}{2}\Big) \cdot 2(x^2 - 2) \\ &= (x-1)(2x+1)\big(x - \sqrt{2}\big)\big(x + \sqrt{2}\big), \end{aligned}$$

and the zeros of f are 1, $-\frac{1}{2}$, $-\sqrt{2}$, $\sqrt{2}$.

5 Multiply by 2: $2x^3 + 3x^2 + 6x - 4 = 0$. Possible rational zeros of the polynomial are $\pm 1, \pm 2, \pm 4, \pm\frac{1}{2}$. Synthetic division shows $\frac{1}{2}$ to be a zero:

$$\begin{array}{c|ccc|c} \frac{1}{2} & 2 & 3 & 6 & -4 \\ & & 1 & 2 & 4 \\ \hline & 2 & 4 & 8 & 0 \end{array}$$

Thus the polynomial factors as $(x - \frac{1}{2})(2x^2 + 4x + 8)$. Equation becomes:

$$(x - \frac{1}{2})(2x^2 + 4x + 8) = 0.$$

Since

$$2x^2 + 4x + 8 = 2(x^2 + 2x + 4) = 2[(x^2 + 2x + 1) + 3] = 2[(x + 1)^2 + 3],$$

$2x^2 + 4x + 8$ has no real zeros, and so $\frac{1}{2}$ is the only real-valued solution to the original equation.

6 Another zero must be $3+2i$ since the polynomial functions has real coefficients. This means that

$$[x - (3 - 2i)][x - (3 + 2i)] = x^2 - 6x + 13$$

is a factor of $f(x)$. With long division we have

$$\frac{f(x)}{x^2 - 6x + 13} = x^2 - 3x - 10,$$

and so $f(x) = (x^2-6x+13)(x^2-3x-10)$. The other zeros of f must be the zeros of $x^2-3x-10$. Since $x^2 - 3x - 10 = (x - 5)(x + 2)$ has zeros -2, 5, the zeros of f are $3 - 2i$, $3 + 2i$, -2, 5.

7a Domain of Q is

$$\{x : x^2 + x - 6 \neq 0\} = \{x : x \neq -3, 2\} = (-\infty, -3) \cup (-3, 2) \cup (2, \infty).$$

7b The only intercept of Q is $(0, 0)$.

7c $x = -3$ and $x = 2$.

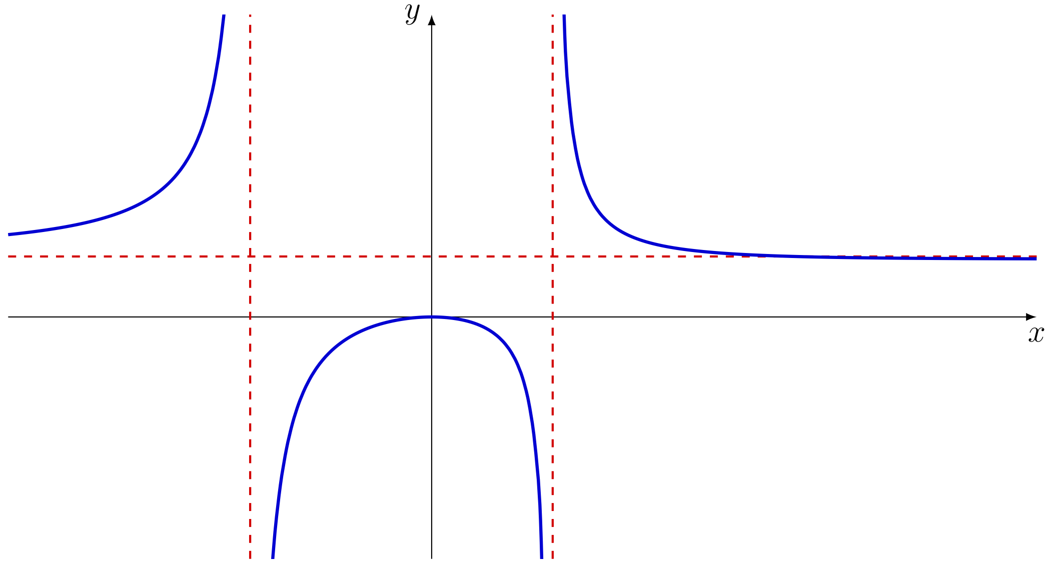
7d $y = 1$ is the horizontal asymptote. No oblique asymptote.

7e Find any x for which $Q(x) = 1$:

$$\frac{x^2}{x^2 + x - 6} = 1 \quad \Rightarrow \quad x^2 = x^2 + x - 6 \quad \Rightarrow \quad x = 6.$$

Thus the graph of Q crosses the horizontal asymptote at $(6, 1)$.

7f Graph:



8a We have

$$3x^3 + 15x^2 < 0 \quad \Rightarrow \quad 3x^2(x+5) < 0 \quad \Rightarrow \quad x + 5 < 0 \quad \Rightarrow \quad x < -5,$$

so $x \in (-\infty, -5)$.

8b For $x \neq 2$ we may divide by $(x-2)^2$ to obtain:

$$\frac{1}{x^2 - 1} > 0 \quad \Rightarrow \quad x^2 - 1 > 0 \quad \Rightarrow \quad x^2 > 1 \quad \Rightarrow \quad |x| > 1,$$

and so either $x > 1$ or $x < -1$. Noting that $x = 2$ satisfies the inequality as well, the solution set is $(-\infty, -1) \cup (1, \infty)$.

9 The domain of f consists of all x for which $(x-2)/(x+4) \geq 0$. We have $(x-2)/(x+4) \geq 0$ if $x - 2 \geq 0$ and $x + 4 > 0$, which occurs if and only if $x \geq 2$. We also have $(x-2)/(x+4) \geq 0$ if $x - 2 \leq 0$ and $x + 4 < 0$, which occurs if and only if $x \leq -4$. Thus

$$\operatorname{Dom} f = \left\{ x : \frac{x-2}{x+4} \geq 0 \right\} = (-\infty, -4) \cup [2, \infty).$$

10a We have $(f \circ g)(x) = f(g(x)) = f(1 - 2x) = \sqrt{(1-2x) - 2} = \sqrt{-2x - 1}$. Domain:

$$\begin{aligned} \operatorname{Dom} f \circ g &= \big\{ x : x \in \operatorname{Dom} g \;\; \& \;\; g(x) \in \operatorname{Dom} f \big\} \\ &= \{ x : x \in \mathbb{R} \;\; \& \;\; 1 - 2x \in [2, \infty) \} \\ &= \{ x : 1 - 2x \geq 2 \} = \big(-\infty, -\tfrac{1}{2} \big]. \end{aligned}$$

10b We have $(f \circ f)(x) = f(f(x)) = f(\sqrt{x-2}) = \sqrt{\sqrt{x-2}-2}$. Domain:

$$\begin{aligned}\operatorname{Dom} f \circ f &= \big\{x : x \in \operatorname{Dom} f \ \ \& \ \ f(x) \in \operatorname{Dom} f\big\} \\ &= \big\{x : x \in [2,\infty) \ \ \& \ \ \sqrt{x-2} \in [2,\infty)\big\} \\ &= \{x : x \geq 2 \ \ \& \ \ x \geq 6\} = [6,\infty).\end{aligned}$$

11a Solve $y = x^2 + 25$ for x, noting that $x \leq 0$ is given:

$$x^2 = y - 25 \quad \Rightarrow \quad |x| = \sqrt{y-25} \quad \Rightarrow \quad -x = \sqrt{y-25} \quad \Rightarrow \quad x = -\sqrt{y-25},$$

and so $f^{-1}(y) = -\sqrt{y-25}$. We can also write this as $f^{-1}(x) = -\sqrt{x-25}$.

11b Solve $y = (3x+2)/(2x-9)$ for x to get

$$x = \frac{9y+2}{2y-3}.$$

Thus

$$f^{-1}(y) = \frac{9y+2}{2y-3}.$$