Math 125 Exam #1 Key (Summer 2017)

1 $f(-2) = \sqrt{(-2)^2 + (-2)} = \sqrt{2}$ and $f(3x) = \sqrt{(3x)^2 + (3x)} = \sqrt{9x^2 + 3x}$.

2a Domain of g: $(-\infty, -3/2) \cup (-3/2, \infty)$.

2b Domain of h: $[4, 7) \cup (7, \infty)$.

3a We have

$$(f+g)(x) = \left(1 + \frac{1}{x}\right) + \frac{1}{x+3} = \frac{x^2 + 5x + 3}{x(x+3)},$$

with domain $\{x : x \neq -3, 0\}$.

3b We have

$$(f/g)(x) = \frac{1 + \frac{1}{x}}{\frac{1}{x+3}} = x + 3 + \frac{x+3}{x} = x + 4 + \frac{3}{x},$$

with domain $\{x : x \neq -3, 0\}$.

4a Since $p(-1) = -3(-1)^2 + 5(-1) = -8 \neq 2$, the point $(-1, 2)$ is not on the graph of p.

4b $p(x) = -2$ implies $-3x^2 + 5x = -2$, which solves to give $x = -\frac{1}{3}, 2$.

4c Domain of p is $(-\infty, \infty)$.

4d Since $p(0) = 0$, the y-intercept of p is $(0, 0)$.

4e We solve $p(x) = 0$, which yields $x = 0, \frac{5}{3}$.

5 Definition of f:

$$f(x) = \begin{cases} x, & x \in [-3, 0] \\ -\frac{3}{4}x + 4, & x \in (0, 4) \end{cases}$$

6a The rectangle's area is the sum of the areas of eight right triangles with legs of length x and $y = \sqrt{4 - x^2}$, and hypotenuse of length 2.

$$A(x) = 8\left(\tfrac{1}{2}x\sqrt{4 - x^2}\right) = 4x\sqrt{4 - x^2}.$$

6b Perimeter is $p(x) = 4x + 4y = 4x + 4\sqrt{4 - x^2}$.

7 Find the solutions to $f(x) = g(x)$, which becomes $(x+6)(x-4) = 0$, and hence $x = -6, 4$. Points of intersection are thus $(-6, f(-6)) = (-6, 3)$ and $(4, f(4)) = (4, 33)$.

8 The zeros of H are

$$H(x)=0 \;\Rightarrow\; 3x^2+x-\frac{1}{2}=0 \;\Rightarrow\; \left(x+\frac{1}{6}\right)^2=\frac{7}{36} \;\Rightarrow\; x=-\frac{1}{6}\pm\frac{\sqrt{7}}{6},$$

and so the x-intercepts are $\left(-\frac{1}{6}\pm\frac{\sqrt{7}}{6},0\right)$.

9 We have

$$2x^2-5x-3\le 0 \;\Rightarrow\; (2x+1)(x-3)\le 0,$$

and so we have either $x\ge -\frac{1}{2}$ and $x\le 3$, or we have $x\le -\frac{1}{2}$ and $x\ge 3$. The latter case is impossible, and so the solution set is $[-\frac{1}{2},3]$.

10 The only fitting quadratic function is

$$f(x)=\frac{75}{200^2}(x-200)(x+200)+75.$$

Thus the height of the cables at the points 100 meters to either side of the center is

$$f(100)=75\left[\frac{(-100)(300)}{200^2}+1\right]=18.75\text{ m}.$$

11 Write $|1-2z|=3$, so either $1-2z=3$ or $1-2z=-3$. Solution set: $\{-1,2\}$.

12 Write $|x-1|<3$, which becomes $-3<x-1<3$. Solution set: $(-2,4)$.