Math 125 Exam #2 Key (Spring 2022)

1 Put the axis of symmetry atop the y-axis such that the vertex is at $(0, 25)$ and the x-intercepts are ± 60. The vertex form for the parabola is $f(x) = ax^2 + 25$, but with $f(60) = 0$ we find that $a = -\frac{1}{144}$, and so $f(x) = -\frac{1}{144}x^2 + 25$. The height of the arch 10 meters from the center is $f(10) = \frac{875}{36} = 24.30\bar{5}$ meters.

2 Solve $f(x) = 0$, or $-2x^2 + 8x + 1 = 0$. With the quadratic forumula we obtain $x = 2 \pm \frac{3\sqrt{2}}{2}$.

3 Write $|x/2| = 2$, so $x/2 = \pm 2$ and finally $x = \pm 4$.

4 From $|1 - 4x| < 6$ we have $-6 < 1 - 4x < 6$, which becomes $-\frac{5}{4} < x < \frac{7}{4}$. Solution set is $(-\frac{5}{4}, \frac{7}{4})$.

5 We have $f(x) = Cx^2(x-2)(x+1)^2$, with $4 = f(1) = C(-1)(2)^2$ implying that $C = -1$. Thus the polynomial function is $f(x) = -x^2(x-2)(x+1)^2$.

6 To have real coefficients the Conjugate Zeros Theorem implies that $2 - i$ must also be a zero, and so we need

$$\begin{aligned} f(x) &= (x+4)[x-(2+i)][x-(2-i)] \\ &= (x+4)(x^2-4x+5) \\ &= x^3 - 11x + 20. \end{aligned}$$

7 The rational zeros that G could possibly have include such values as 1 and -5, which are in fact zeros for G. We use synthetic division to start factoring $G(x)$:

$$\begin{array}{r|rrrr|r} 1 & 2 & 11 & -5 & -43 & 35 \\ & & 2 & 13 & 8 & -35 \\ \hline & 2 & 13 & 8 & -35 & 0 \end{array} \quad \longrightarrow \quad f(x) = (x-1)(2x^3 + 13x^2 + 8x - 35)$$

$$\begin{array}{r|rrr|r} -5 & 2 & 13 & 8 & -35 \\ & & -10 & -15 & 35 \\ \hline & 2 & 3 & -7 & 0 \end{array} \quad \longrightarrow \quad f(x) = (x-1)(x+5)(2x^2 + 3x - 7)$$

Solving $2x^2 + 3x - 7 = 0$ using the quadratic formula, we obtain the complete list of real zeros: -5, 1, $\frac{-3 \pm \sqrt{65}}{4}$.

The complete factorization is

$$G(x) = (x-1)(x+5)\left(x + \frac{3+\sqrt{65}}{4}\right)\left(x + \frac{3-\sqrt{65}}{4}\right).$$

8 Let $f(x) = x^3 - 8x^2 + 25x - 26$, so the problem is to find all x such that $f(x) = 0$. Among the possible rational zeros is 2, which turns out to work:

$$\begin{array}{r|rrr|r} 2 & 1 & -8 & 25 & -26 \\ & & 2 & -12 & 26 \\ \hline & 1 & -6 & 13 & 0 \end{array} \quad \longrightarrow \quad \text{So 2 is a zero and } f(x) = (x-2)(x^2 - 6x + 13).$$

Applying ye olde quadratic formula to $x^2 - 6x + 13 = 0$ yields the zeros $3 \pm 2i$. In conclusion, f has zeros 2, $3 - 2i$, $3 + 2i$, which are therefore the solutions to the given equation.

9 Factoring shows the fraction is reduced:

$$\Psi(x) = \frac{x(x^2+2)}{(x-3)(x-4)}.$$

There are vertical asymptotes $x = 3$, $x = 4$. Also, long division also shows $y = x + 7$ is an oblique asymptote:

$$\begin{array}{r|l} & x \quad + 7 \\ \hline x^2 - 7x + 12 & x^3 \qquad\quad + 2x \\ & -x^3 + 7x^2 - 12x \\ \hline & \qquad 7x^2 - 10x \\ & \qquad -7x^2 + 49x - 84 \\ \hline & \qquad\qquad 39x - 84 \end{array}$$

10a Write $x^3 + x^2 - 4x - 4 < 0$, then factor by grouping to get $(x+1)(x-2)(x+2) < 0$. Using the Intermediate Value Theorem we find the solution set to be $(-\infty, -2) \cup (-1, 2)$.

10b Write

$$\frac{2x-6}{1-x} - \frac{2(1-x)}{1-x} \le 0 \quad \hookrightarrow \quad \frac{4x-8}{1-x} \le 0 \quad \hookrightarrow \quad f(x) = \frac{x-2}{1-x} \le 0.$$

Here f has zero 2 and vertical asymptote $x = 1$. Choose test values in the intervals $(-\infty, 1)$, $(1, 2)$, and $(2, \infty)$, and use the Intermediate Value Theorem to find the solution set to be $(-\infty, 1) \cup [2, \infty)$.

11 Find all x such that $x^3 > 4x^2$, which becomes $x^2(x-4) > 0$. Since $x^2 > 0$ for any $x \neq 0$, the inequality is satisfied if and only if $x - 4 > 0$. Solution set is $(4, \infty)$.