Math 125 Exam #1 Key (Spring 2022)

1 $f(-3) = -2$ and $f(x-2) = 1 - \dfrac{45}{(x-2)^2+6}$.

2a $D_\ell = \{x \mid x^2+3x+2 \neq 0\} = \{x \mid x \neq -1, -2\} = (-\infty, -2) \cup (-2, -1) \cup (-1, \infty)$.

2b $D_u = \{t \mid t+8 > 0\} = (-8, \infty)$.

3 First,

$$(f/g)(x) = \frac{2 - \dfrac{1}{x-9}}{\dfrac{\sqrt{x}}{x-1}} = \left(2 - \frac{1}{x-9}\right)\left(\frac{x-1}{\sqrt{x}}\right)$$

Now, $D_f = \{x \mid x \neq 9\}$, $D_g = [0,1) \cup (1, \infty)$, and $g(x) = 0$ only if $x = 0$. With these facts we find

$$D_{f/g} = \{x \mid x \in D_f \cap D_g \;\; \& \;\; g(x) \neq 0\} = (0,1) \cup (1,9) \cup (9, \infty).$$

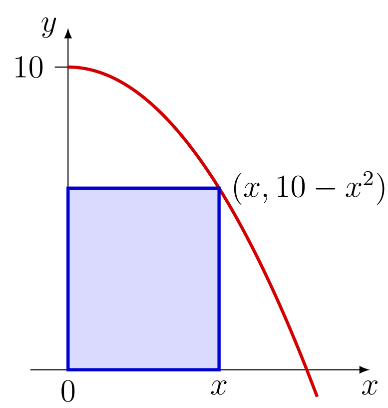
4 $-1 = f(2) = \dfrac{5}{6-c}$, and so $c = 11$.

5 $D_f = [-5, 5)$ and $R_f = (0, 5] \cup \{-3\}$. Definition follows:

$$f(x) = \begin{cases} x+7, & -5 \leq x < -2 \\ -\frac{5}{2}x, & -2 \leq x < 0 \\ -3, & 0 \leq x < 5 \end{cases}$$

6 $y = x^2 \xrightarrow{(1)} y = (x-3)^2 \xrightarrow{(2)} y = (-x-3)^2 \xrightarrow{(3)} f(x) = (-x-3)^2 - 4$.

7 $A(x) = x(10 - x^2)$, and since the x-intercept of the parabola in Quadrant I is $\sqrt{10}$, the domain of A must be $(0, \sqrt{10})$. See figure below.



8 Solving $2x^2 + 5x + 3 = 0$, we get $x^2 + \frac{5}{2}x + (\frac{5}{4})^2 = -\frac{3}{2} + (\frac{5}{4})^2$, or $(x + \frac{5}{4})^2 = \frac{1}{16}$, which upon taking the square root yields $x = -\frac{5}{4} \pm \frac{1}{4}$. The zeros of p are $-\frac{3}{2}$ and -1.

9 From $u(x) = 0$ factorization gives $(x^2 - 4)(x^2 - 6) = 0$, and so the zeros of u are $\pm 2, \pm\sqrt{6}$.

10 The vertex is at $(-\frac{1}{3}, \frac{14}{3})$, with axis of symmetry $x = -\frac{1}{3}$, domain $D_f = (-\infty, \infty)$, and range $R_f = [\frac{14}{3}, \infty)$.