MATH 125 EXAM #1 KEY (SPING 2021)

1 $f(-3) = \sqrt{6}$ and $f(x+1) = \sqrt{(x+1)^2 + (x+1)}$.

2a Domain of q is $(8, \infty)$.

2b Domain of h is $\{x : x \geq 0 \text{ and } x \neq \frac{2}{5}\} = [0, \frac{2}{5}) \cup (\frac{2}{5}, \infty)$

3a We have

$$(f \cdot g)(x) = \left(1 + \frac{1}{2x}\right)\left(\frac{x+2}{x+4}\right),$$

with domain $\{x : x \neq -4, 0\}$.

3b We have

$$(f/g)(x) = \frac{1 + \frac{1}{2x}}{\frac{x+2}{x+4}} = \left(1 + \frac{1}{2x}\right)\left(\frac{x+4}{x+2}\right),$$

with domain $\{x : x \neq -4, -2, 0\}$.

4a Since $f(-x) = (-x)^3 - 10 = -x^3 - 10 \neq \pm f(x)$, f is neither.

4b Since $g(-x) = \dfrac{(-x)^3}{3(-x)^5 - 9(-x)} = \dfrac{x^3}{3x^5 - 9x} = g(x)$, g is even.

5 Definition of f:

$$f(x) = \begin{cases} -2x - 6, & x \in [-5, -2] \\ x, & x \in (-2, 0) \\ -x/5 + 4, & x \in [0, 5] \end{cases}$$

6a Cut the wire into lengths $4x$ and $10 - 4x$. With the $4x$ piece make a square with sides of length x, and with the $10 - 4x$ piece make a circle. The area of the square is x^2, and the area of the circle (which has circumference $10 - 4x$) is $(5 - 2x)^2/\pi$. Total area is

$$A(x) = \frac{(5 - 2x)^2}{\pi} + x^2.$$

6b Domain of A is $\{x : 0 < 4x < 10\}$, or $\left(0, \frac{5}{2}\right)$.

7 Solve $G(x) = 0$, or $x^2 + 8x + 12 = 0$, by factoring to get the zeros -6 and -2, which are also the x-intercepts.

8 Solve $H(x) = 0$, which becomes $(x^3 - 8)(x^3 - 1) = 0$, and hence

$$(x - 2)(x^2 + 2x + 4)(x - 1)(x^2 + x + 1) = 0.$$

Thus $x = 1$ and $x = 2$. But also $x^2 + 2x + 4 = 0$ gives $x = -1 \pm i\sqrt{3}$, and $x^2 + x + 1 = 0$ gives $x = -\frac{1}{2} \pm i\frac{\sqrt{3}}{2}$. Set of zeros are: $\left\{1, 2, -1 \pm i\sqrt{3}, -\frac{1}{2} \pm i\frac{\sqrt{3}}{2}\right\}$.

9 Consider the equation $2x^2 = 3 + 5x$. Write as $2x^2 - 5x - 3 = 0$, and let $f(x) = 2x^2 - 5x - 3$ so the equation becomes $f(x) = 0$. The solutions are $x = -\frac{1}{2}$ and $x = 3$, which are thus the x-intercepts of f. But f graphs as a parabola that opens upward, with vertex *below* the x-axis at $(\frac{5}{4}, -\frac{49}{8})$. Since the axis of symmetry of the parabola lies between the x-intercepts and includes the vertex (the parabola's lowest point), it must be that $f(x) < 0$ for all x between $-\frac{1}{2}$ and 3. That is, the inequality $2x^2 < 3 + 5x$ has solution set consisting of the interval $(-\frac{1}{2}, 3)$.

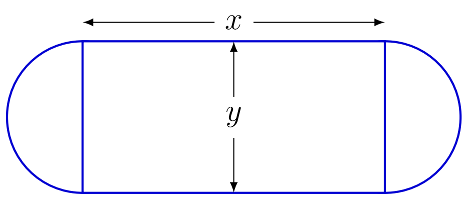
10 Let x be the length of the rectangle and y the width, as shown below. Semicircles of diameter y are attached to the width sides of the rectangle. The two semicircles together have the circumference of a whole circle of diameter y, which we know is πy. The inside perimeter of the track is thus $2x + \pi y = 1500$, and hence $y = \frac{1500-2x}{\pi}$. Now, the area of the rectangle is $A = xy$, or, as a function of x,

$$A(x) = x\left(\frac{1500 - 2x}{\pi}\right) = -\frac{2}{\pi}x^2 + \frac{1500}{\pi}x.$$

This graphs as a parabola that opens downward with vertex at

$$x = -\frac{b}{2a} = -\frac{1500}{\pi}\left(-\frac{\pi}{4}\right) = 375.$$

Dimensions of the rectangle, in meters, should thus be $375 \times 750/\pi$ to maximize area.



11 Write $|3t - 4| = 5$, so either $3t - 4 = 5$ or $3t - 4 = -5$. Solution set: $\{-\frac{1}{3}, 3\}$.

12 Write $|x+5| > 15$, so either $x+5 > 15$ or $x+5 < -15$. Solution set: $(-\infty, -20) \cup (10, \infty)$.