Math 125 Exam #3 Key (Spring 2020)

1 Domain is $(-\infty, \infty)$ and range is $(-\infty, 6)$. Horizontal asymptote: $y = 6$.

2 Equation becomes $3^{3x^2+4x} = 3^{-1}$, and so $3x^2 + 4x = -1$. This solves to give solution set $\{-\frac{1}{3}, -1\}$.

3 Domain is

$$\{x : 4 - x/3 > 0\} = (-\infty, 12).$$

4 Let $y = f(x)$, so $y = \frac{1}{2}\log(2x) - 9$, giving $\log(2x) = 2y + 18$, and then $10^{2y+18} = 2x$. Solving for x gives $x = \frac{1}{2} \cdot 10^{2y+18}$, and therefore $f^{-1}(y) = \frac{1}{2} \cdot 10^{2y+18}$.

5 From $e^{-2x+3} = 12$ we get $-2x + 3 = \ln 12$, and hence

$$x = \frac{3 - \ln 12}{2}.$$

6 With laws of logarithms:

$$\frac{4}{3}\ln(x-4) - \frac{2}{3}\ln(x^2 - 1).$$

7 Since the logarithm function is one-to-one, we have

$$\ln y = \ln\left(\frac{Cx^2}{x+1}\right) \quad \Rightarrow \quad y = \frac{Cx^2}{x+1}.$$

8a From $\log_5(x+3)(x-1) = 1$ we obtain

$$(x+3)(x-1) = 5 \quad \Rightarrow \quad x^2 + 2x - 8 = 0 \quad \Rightarrow \quad x = -4, 2.$$

However, -4 is an extraneous solution, so the solution set is $\{2\}$.

8b Take the natural logarithm of both sides to get the exact solution:

$$(1+x)\ln 0.3 = (2x-1)\ln 1.7 \quad \Rightarrow \quad x = \frac{\ln 1.7 + \ln 0.3}{2\ln 1.7 - \ln 0.3}.$$

8c With the Change-of-Base Formula we get

$$\log_2(x+1) - \frac{\log_2 x}{\log_2 4} = 1 \quad \Rightarrow \quad \log_2(x+1) - \frac{1}{2}\log_2 x = 1 \quad \Rightarrow \quad \log_2 \frac{x+1}{\sqrt{x}} = 1,$$

giving

$$\frac{x+1}{\sqrt{x}} = 2 \quad \Rightarrow \quad x - 2\sqrt{x} + 1 = 0 \quad \Rightarrow \quad (\sqrt{x} - 1)^2 = 0 \quad \Rightarrow \quad x = 1.$$

Solution set is $\{1\}$.

9a $N(t) = N_0 e^{kt}$.

9b Let $t = 0$ for the year 2005, so $N_0 = N(0) = 900,000$, and then $N(t) = 900,000e^{kt}$. Using $N(2) = 800,000$ gives

$$800,000 = 900,000e^{2k} \quad \Rightarrow \quad k = \tfrac{1}{2}\ln\tfrac{8}{9},$$

so

$$N(t) = 900,000e^{(t/2)\ln(8/9)} = 900,000\left(\frac{8}{9}\right)^{t/2}.$$

Population in 2009 is thus

$$N(4) = 900,000\left(\frac{8}{9}\right)^2 \approx 711,111.$$

10 We're given $u_0 = 100\,^\circ\mathrm{C}$ (so $u(0) = 100$), $T = 20\,^\circ\mathrm{C}$, and also $u(15) = 75$. With Newton's Law of Cooling:

$$75 = u(15) = 20 + (100 - 20)e^{15k},$$

which gives $k = \frac{1}{15}\ln\frac{55}{80} \approx -0.02498$. Thus the model is

$$u(t) = 20 + 80e^{-0.02498t}.$$

After another 10 minutes (when $t = 25$), the temperature is

$$u(25) = 20 + 80e^{-0.02498(25)} \approx 62.8\,^\circ\mathrm{C}.$$

11 Convert 0.117° to minutes:

$$(0.117^\circ)\left(\frac{60'}{1^\circ}\right) = 7.02'.$$

Convert $0.02'$ to seconds, and round to the nearest second:

$$(0.02')\left(\frac{60''}{1'}\right) = 1.2'' \approx 1''.$$

Therefore $127.117^\circ \approx 127^\circ\, 7'\, 1''$.

12 The point given lies on a circle of radius $\frac{1}{2}$, so:

$$\sin\theta = \frac{0.4}{1/2} = \frac{4}{5}, \quad \cos\theta = \frac{-0.3}{1/2} = -\frac{3}{5}, \quad \tan\theta = -\frac{4}{3}, \quad \csc\theta = \frac{5}{4}, \quad \sec\theta = -\frac{5}{3}, \quad \cot\theta = -\frac{3}{4}.$$

13 $\tan\theta = \sin\theta/\cos\theta = \frac{1}{2}$, $\cot\theta = 2$, $\csc\theta = -\sqrt{5}$, $\sec\theta = -\frac{\sqrt{5}}{2}$.

14 Straight away we have $\sin\theta = \frac{1}{3}$. Now, $\csc\theta > 0$ implies θ is in quadrant QI or QII, while $\cot\theta < 0$ implies θ is in QII or QIV. Thus θ is in QII, and since $\cos\theta < 0$ in QII, we have

$$\cos^2\theta = 1 - \sin^2\theta \quad \Rightarrow \quad \cos\theta = -\sqrt{1-\sin^2\theta} = -\sqrt{1-(1/3)^2} = -\frac{2\sqrt{2}}{3}.$$

The rest come easily:

$$\tan\theta = \frac{1/3}{-2\sqrt{2}/3} = -\frac{1}{2\sqrt{2}}, \quad \cot\theta = \frac{1}{\tan\theta} = -2\sqrt{2}, \quad \sec\theta = \frac{1}{\cos\theta} = -\frac{3}{2\sqrt{2}}.$$