

MATH 125 EXAM #4 KEY (SPRING 2019)

1a $-\pi/5$

1b Undefined.

1c $\sqrt{5}/2$

1d We have

$$\theta = \cos^{-1}\left(\sin \frac{7\pi}{6}\right) \Rightarrow \cos \theta = \sin \frac{7\pi}{6} = \cos\left(\frac{\pi}{2} - \frac{7\pi}{6}\right) = \cos \frac{2\pi}{3} \Rightarrow \theta = \frac{2\pi}{3}.$$

2a We have $\sin \theta = \pm \frac{1}{\sqrt{2}}$, and so $\theta = \frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}$.

2b So $\tan \theta = -1$, giving $\theta = \frac{3\pi}{4}, \frac{7\pi}{4}$.

2c Write $4 + 4 \sin \theta = 1 - \sin^2 \theta$, so $(\sin \theta + 3)(\sin \theta + 1) = 0$, which implies $\sin \theta = -1$, or $\theta = \frac{3\pi}{2}$.

3a We have

$$1 - \frac{\sin^2 \theta}{1 - \cos \theta} = 1 - \frac{1 - \cos^2 \theta}{1 - \cos \theta} = 1 - \frac{(1 - \cos \theta)(1 + \cos \theta)}{1 - \cos \theta} = 1 - (1 + \cos \theta) = -\cos \theta.$$

3b We have

$$\sin \theta \tan \theta = \frac{\sin^2 \theta}{\cos \theta} = \frac{1 - \cos^2 \theta}{\cos \theta} = \frac{1}{\cos \theta} - \frac{\cos^2 \theta}{\cos \theta} = \sec \theta - \cos \theta.$$

4 The expression becomes $\cos\left(\frac{5\pi}{12} + \frac{7\pi}{12}\right) = \cos \pi = -1$.

5 With a half-angle identity,

$$\cos 22.5^\circ = \sqrt{\frac{1 + \cos 45^\circ}{2}} = \sqrt{\frac{1 + 1/\sqrt{2}}{2}} = \frac{\sqrt{2 + \sqrt{2}}}{2}.$$

6 $\cos^4 \theta - \sin^4 \theta = (\cos^2 \theta - \sin^2 \theta)(\cos^2 \theta + \sin^2 \theta) = \cos^2 \theta - \sin^2 \theta = \cos(2\theta).$

7 This sets up a right triangle with legs of length 12 and 3, and so if the angle of depression is θ , we have

$$\tan \theta = -\frac{3}{12} \Rightarrow \theta = \tan^{-1}(-0.25) = -14.0^\circ.$$

8 Let h be the height of the monument. Then

$$\tan 35.1^\circ = \frac{h}{789} \Rightarrow h = 789 \tan 35.1^\circ \approx 554.5 \text{ ft}$$

9a $B = 110^\circ$ is immediate, and with the Law of Sines we find that $b = 3.68$ and $c = 1.34$.

9b Use the Law of Sines again to get one triangle with $a = 6.21$, $A = 86.5^\circ$, $C = 53.5^\circ$, and another triangle with $a = 1.45$, $A = 13.5^\circ$, $C = 126.5^\circ$.

9c Use Law of Cosines twice to get $A = 33.6^\circ$, $B = 62.2^\circ$, $C = 84.3^\circ$.

10 Use the Law of Cosines to find that the guy wires should be 520.1 ft and 499.5 ft.