Math 125 Exam #1 Key (Spring 2019)

1 $f(-3)=\sqrt{(-3)^2-(-3)}=\sqrt{12}=2\sqrt{3}$ and $f(-2x)=\sqrt{(-2x)^2-(-2x)}=\sqrt{4x^2+2x}$.

2a Domain of g: $(-\infty,\frac{3}{4})\cup(\frac{3}{4},\infty)$.

2b Domain of h: $[4,7)\cup(7,\infty)$.

3a We have

$$(f+g)(x)=\left(1+\frac{1}{x}\right)+\frac{1}{x+3}=\frac{x^2+5x+3}{x(x+3)},$$

with domain $\{x : x\neq -3,0\}$.

3b We have

$$(f/g)(x)=\frac{1+\frac{1}{x}}{\frac{1}{x+3}}=x+3+\frac{x+3}{x}=x+4+\frac{3}{x},$$

with domain $\{x : x\neq -3,0\}$.

4a Since $p(-1)=-3(-1)^2+5(-1)=-8\neq 2$, the point $(-1,2)$ is not on the graph of p.

4b $p(x)=-2$ implies $-3x^2+5x=-2$, which solves to give $x=-\frac{1}{3},2$.

4c Domain of p is $(-\infty,\infty)$.

4d Since $p(0)=0$, the y-intercept of p is $(0,0)$.

4e We solve $p(x)=0$, which yields $x=0,\frac{5}{3}$.

5 Definition of f:

$$f(x)=\begin{cases} x, & x\in[-3,0] \\ -\frac{3}{4}x+4, & x\in(0,4) \end{cases}$$

6a The rectangle's area is the sum of the areas of eight right triangles with legs of length x and $y=\sqrt{4-x^2}$, and hypotenuse of length 2.

$$A(x)=8\left(\tfrac{1}{2}x\sqrt{4-x^2}\right)=4x\sqrt{4-x^2}.$$

6b Perimeter is $p(x)=4x+4y=4x+4\sqrt{4-x^2}$.

7 Find the solutions to $f(x)=g(x)$, which becomes $(x+6)(x-4)=0$, and hence $x=-6,4$. Points of intersection are thus $(-6,f(-6))=(-6,3)$ and $(4,f(4))=(4,33)$.

8 The zeros of H are

$$H(x) = 0 \quad \Rightarrow \quad 3x^2 + x - \frac{1}{2} = 0 \quad \Rightarrow \quad \left(x + \frac{1}{6}\right)^2 = \frac{7}{36} \quad \Rightarrow \quad x = -\frac{1}{6} \pm \frac{\sqrt{7}}{6},$$

and so the x-intercepts are $\left(-\frac{1}{6} \pm \frac{\sqrt{7}}{6}, 0\right)$.

9 We have

$$x^2 + 5x + 6 \geq 0 \quad \Rightarrow \quad (x+2)(x+3) \geq 0,$$

implying either $x \geq -2$ or $x \leq -3$, and so the solution set is $(-\infty, -3) \cup (-2, \infty)$.

10 The only fitting quadratic function is

$$f(x) = \frac{75}{200^2}(x - 200)(x + 200) + 75.$$

Thus the height of the cables at the points 100 meters to either side of the center is

$$f(100) = 75\left[\frac{(-100)(300)}{200^2} + 1\right] = 18.75 \text{ m}.$$

11 Write $|4 - 5t| = 19$, so either $4 - 5t = 19$ or $4 - 5t = -19$. Solution set: $\{-3, \frac{23}{5}\}$.

12 Write $|y - 1| > 7$, so either $y - 1 < -7$ or $y - 1 > 7$. Solution set: $(-\infty, -6) \cup (8, \infty)$.