Math 125 Exam #4 Key (Spring 2014)

1a $(-340^\circ)\left(\frac{\pi}{180^\circ}\right) = -\frac{17}{9}\pi.$

1b $(8\pi)\frac{180^\circ}{\pi} = 1440^\circ$

2 Amplitude is $1/2$, period is $2\pi/3$. If we write

$$-\frac{1}{2}\sin\left[3\left(x - \frac{\pi}{6}\right)\right]$$

we can see that the phase shift is $\pi/6$.

3 We have

$$\frac{\tan^2\theta}{\sec\theta} \div \frac{3\tan^3\theta}{\sec\theta} = \frac{\tan^2\theta}{\sec\theta}\cdot\frac{\sec\theta}{3\tan^3\theta} = \frac{1}{3\tan\theta} = \frac{1}{3}\cot\theta.$$

4 With a half-angle identity we obtain

$$\tan 105^\circ = \tan(60^\circ + 45^\circ) = \frac{\tan 60^\circ + \tan 45^\circ}{1 - \tan 60^\circ \tan 45^\circ} = \frac{\sqrt{3}+1}{1-\sqrt{3}} = -2-\sqrt{3}.$$

5a $\dfrac{1+\cos^2 x}{\sin^2 x} = \dfrac{1+(1-\sin^2 x)}{\sin^2 x} = \dfrac{2}{\sin^2 x} - 1 = 2\csc^2 x - 1.$

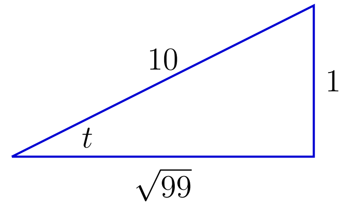
5b We have

$$\begin{aligned}\frac{1+\sin x}{1-\sin x} &= \frac{1+\sin x}{1-\sin x}\cdot\frac{1+\sin x}{1+\sin x} = \frac{1+2\sin x+\sin^2 x}{1-\sin^2 x} = \frac{1+2\sin x+\sin^2 x}{\cos^2 x}\\ &= \sec^2 x + 2\tan x\sec x + \tan^2 x = (\sec x + \tan x)^2.\end{aligned}$$

6a $\sin^{-1}\left(\cos\frac{\pi}{6}\right) = \sin^{-1}\left(\frac{\sqrt{3}}{2}\right) = \frac{\pi}{3}$

6b Let $t = \sin^{-1}\left(\frac{1}{10}\right)$, so that $\sin t = \frac{1}{10}$. Thus t is the angle depicted in the triangle below, and so

$$\tan\left(\sin^{-1}\tfrac{1}{10}\right) = \tan t = \frac{1}{\sqrt{99}} = \frac{\sqrt{11}}{33}.$$



7a Divide by 2 and then square both sides to obtain

$$\cos^2 x + 2\sin x\cos x + \sin^2 x = \frac{3}{2}.$$

Since $\cos^2 x + \sin^2 x = 1$, we then get

$$2\cos x\sin x = \frac{1}{2} \quad\Rightarrow\quad \sin 2x = \frac{1}{2} \quad\Rightarrow\quad 2x = \frac{\pi}{6}, \frac{5\pi}{6} \quad\Rightarrow\quad x = \frac{\pi}{12}, \frac{5\pi}{12}.$$

Note: two other solutions to $\sin 2x = \frac{1}{2}$ in $[0, 2\pi)$ are $\frac{13\pi}{6}$ and $\frac{17\pi}{6}$, but these are extraneous solutions; that is, they don't satisfy the original equation. Solution set is $\left\{\frac{\pi}{12}, \frac{5\pi}{12}\right\}$.

7b We have

$$2\sec x\tan x + 2\sec x + \tan x + 1 = (2\sec x)(\tan x + 1) + (\tan x + 1) = (\tan x + 1)(2\sec x + 1) = 0,$$

so that either $\tan x + 1 = 0$ or $2\sec x + 1 = 0$. From the first equation we get $\tan x = -1$, and so $x = \frac{3\pi}{4}, \frac{7\pi}{4}$. The second equation has no solution. Thus the solution set is $\left\{\frac{3\pi}{4}, \frac{7\pi}{4}\right\}$.

8a By the Law of Sines:

$$\sin A = \frac{a\sin C}{c} = \frac{56.2\sin(46°32')}{22.1} \approx 1.85.$$

There is no solution.

8b We have

$$\begin{aligned}\frac{\sin B}{10} = \frac{\sin 10°}{3} \quad &\Rightarrow \quad \sin B = 0.57883\\ &\Rightarrow \quad \sin^{-1}(\sin B) = \sin^{-1}(0.57883) = 35.366°\\ &\Rightarrow \quad \sin B = \sin 35.366°.\end{aligned}$$

One solution to this equation is of course $B_1 = 35.366°$; however B could also be the Quadrant II angle

$$B_2 = 180° - \sin^{-1}(0.57883) = 144.63°$$

(see very pretty picture below).

For the angle B_1 we get $C_1 = 134.634°$, and then by the Law of Cosines we obtain

$$c_1^2 = a^2 + b^2 - 2ab\cos C_1 = 3^2 + 10^2 - 2(3)(10)\cos 134.634° = 151.156 \quad\Rightarrow\quad c_1 = 12.29.$$

So one possible triangle (rounding to the nearest tenth) has

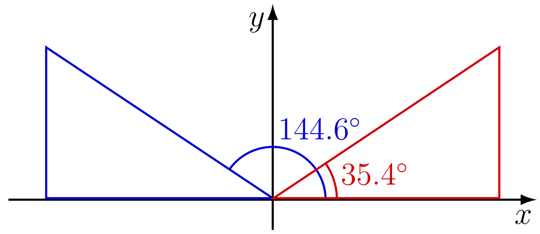
$$B_1 = 35.4°, \quad C_1 = 134.6°, \quad c_1 = 12.3.$$

For the angle B_2 we get $C_2 = 25.370°$, and then by the Law of Cosines we obtain

$$c_2^2 = a^2 + b^2 - 2ab\cos C_2 = 3^2 + 10^2 - 2(3)(10)\cos 25.370° = 54.786 \quad\Rightarrow\quad c_1 = 7.40.$$

So another possible triangle has

$$B_2 = 144.6°, \quad C_2 = 25.4°, \quad c_2 = 7.4.$$



8c The Law of Cosines is necessary here:

$$\begin{aligned} c^2 = a^2 + b^2 - 2ab\cos C \quad &\Rightarrow \quad 6^2 = 4^2 + 3^2 - 2(4)(3)\cos C \\ &\Rightarrow \quad \cos C = -11/24 \\ &\Rightarrow \quad C = \cos^{-1}(-11/24) \approx 117.28^\circ. \end{aligned}$$

And

$$\begin{aligned} b^2 = a^2 + c^2 - 2ac\cos B \quad &\Rightarrow \quad 3^2 = 4^2 + 6^2 - 2(4)(6)\cos B \\ &\Rightarrow \quad \cos B = 43/48 \\ &\Rightarrow \quad B = \cos^{-1}(43/48) \approx 26.38^\circ. \end{aligned}$$

Finally, $A = 180^\circ - 26.38^\circ - 117.28^\circ = 36.34^\circ$. To the nearest tenth we have:

$$A = 36.3^\circ, \quad B = 26.4^\circ, \quad C = 117.3^\circ.$$