Math 125 Exam #3 Key (Spring 2014)

1a We have

$$\text{Dom}(f) = \{x : x^2 - 4 \neq 0\} = \{x : x \neq -2, 2\}.$$

1b The x-intercepts of f are the points $(x, f(x))$ where $f(x) = 0$:

$$\frac{x^2(x+1)}{(x-2)(x+2)} = 0 \quad \Rightarrow \quad x^2(x+1) = 0 \quad \Rightarrow \quad x = -1, 0$$

so $(-1, 0)$ and $(0, 0)$ are the x-intercepts. Since $(0, 0)$ is also a y-intercept of f and a function can never have more than one y-intercept, we have found all intercepts.

1c The vertical asymptotes of f are $x = -2$ and $x = 2$.

1d The degree of the numerator is 1 greater than the degree of the denominator, so there will be an oblique asymptote. From the division

$$\begin{array}{r|l} & \qquad\qquad\quad x + 1 \\ \hline x^2 - 4 & x^3 + x^2 \\ & -x^3 \qquad\; + 4x \\ \hline & \qquad x^2 + 4x \\ & \quad -x^2 \qquad + 4 \\ \hline & \qquad\qquad 4x + 4 \end{array}$$

we find that

$$f(x) = x + 1 + \frac{4x+4}{x^2-4},$$

and therefore $y = x + 1$ is the equation of the oblique asymptote.

1e The graph of f intersects the oblique asymptote $y = x + 1$ if there is some $x \in \text{Dom}(f)$ for which $f(x) = x + 1$. This results in the equation

$$\frac{x^3 + x^2}{x^2 - 4} = x + 1,$$

giving

$$x^3 + x^2 = x^3 + x^2 - 4x - 4 \quad \Rightarrow \quad 4x = -4 \quad \Rightarrow \quad x = -1.$$

Thus the graph of f intersects $y = x + 1$ at $(-1, 0)$.

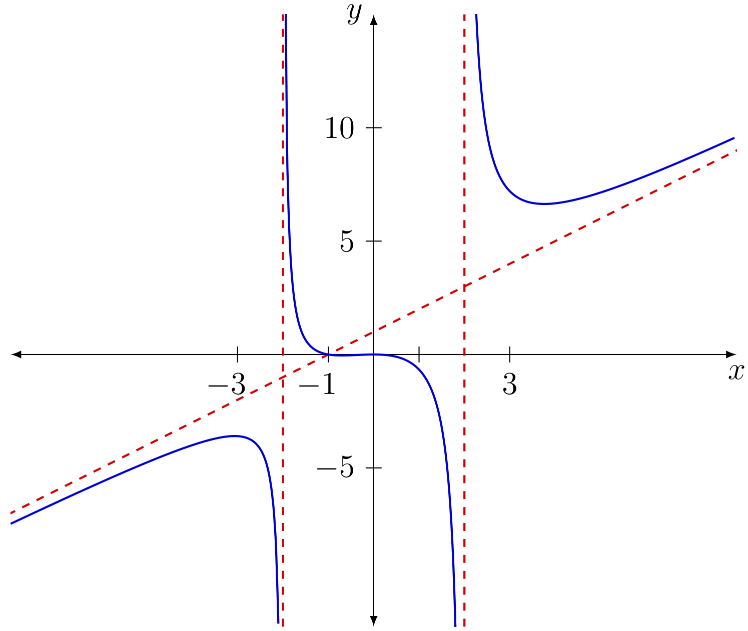
1f The vertical asymptotes partition the plane into three regions:

$$R_1 = \{x : x < -2\}, \quad R_2 = \{x : -2 < x < 2\}, \quad \text{and} \quad R_3 = \{x : x > 2\}.$$

We will want at least one point that lies on the graph of f in each region. We calculate

$$f(-3) = -3\tfrac{3}{5}, \quad f(3) = 7\tfrac{1}{5},$$

we obtain the points $\left(-3, -3\tfrac{3}{5}\right)$ and $\left(3, 7\tfrac{1}{5}\right)$. We sketch the graph:



2a Factoring, we get $(x+2)(x+3) > 0$. There are two cases to consider.

Case 1: $x+2 > 0$ & $x+3 > 0$. This gives $x > -2$ & $x > -3$, which is equivalent to $x > -2$.

Case 2: $x+2 < 0$ & $x+3 < 0$. This gives $x < -2$ & $x < -3$, which is equivalent to $x < -3$.

Thus we may have $x < -3$ or $x > -2$. Solution set: $(-\infty, -3) \cup (-2, \infty)$.

2b Manipulate without multiplying by an expression involving x:

$$\frac{2x+1}{x-5} \le 3 \quad \Leftrightarrow \quad \frac{2x+1}{x-5} - 3 \le 0 \quad \Leftrightarrow \quad \frac{2x+1-3(x-5)}{x-5} \le 0 \quad \Leftrightarrow \quad \frac{16-x}{x-5} \le 0.$$

There are two cases to consider.

Case 1: $16 - x \ge 0$ & $x - 5 < 0$. This gives $x \le 16$ & $x < 5$, which is equivalent to $x < 5$. (Note that we cannot have $x = 5$ since division by zero would result.)

Case 2: $16 - x \le 0$ & $x - 5 > 0$. This gives $x \ge 16$ & $x > 5$, which is equivalent to $x \ge 16$. (Letting $x = 16$ results in no division by zero.)

Thus we may have $x < 5$ or $x \ge 16$. Solution set: $(-\infty, 5) \cup [16, \infty)$.

2c Write as $x^2 - 4x + 12 < 0$, and note that the trinomial cannot be factored. Complete the square:

$$x^2 - 4x + 12 < 0 \quad \Rightarrow \quad (x^2 - 4x + 4) + 8 < 0 \quad \Rightarrow \quad (x-2)^2 + 8 < 0 \quad \Rightarrow \quad (x-2)^2 < -8.$$

Since there is no real number x that makes $(x-2)^2$ negatively valued, there is no solution. Solution set: $\varnothing$.

3a Using the formula provided,

$$A(t) = 750\left(1 + \frac{0.08}{4}\right)^{4t} = 750(1.02)^{4t}.$$

3b $A(20) = 750(1.02)^{80} = \$3656.58$.

4 Using laws of logarithms,

$$\log_2(ab^2)^5 - \log_2(3a^2b) + \log_2(12a^3) = \log_2\left[\frac{(ab^2)^5}{3a^2b}\right] + \log_2(12a^3) = \log_2\left[\frac{(ab^2)^5}{3a^2b} \cdot 12a^3\right],$$

and finally $\log_2(4a^6b^9)$.

5a We have

$$4^{3-2x} = 64 \quad \Rightarrow \quad 4^{3-2x} = 4^3 \quad \Rightarrow \quad 3 - 2x = 3 \quad \Rightarrow \quad x = 0.$$

5b Take the logarithm of each side:

$$\ln(3^x) = \ln(6^{x-1}) \quad \Rightarrow \quad x \ln 3 = (x-1)\ln 6 \quad \Rightarrow \quad x\ln 3 - x \ln 6 = -\ln 6 \quad \Rightarrow \quad x = \frac{\ln 6}{\ln 6 - \ln 3}.$$

5c Multiply by e^x to get $e^{2x} - 12 - e^x = 0$; now,

$$e^{2x} - e^x - 12 = 0 \quad \Rightarrow \quad (e^x - 4)(e^x + 3) = 0 \quad \Rightarrow \quad e^x = 4 \quad \Rightarrow \quad x = \ln 4.$$

(Note that $e^x = -3$ has no solution.)

5d Convert to an exponential equation:

$$\log_2(10 + 3x) = 5 \quad \Rightarrow \quad 2^5 = 10 + 3x \quad \Rightarrow \quad 3x = 22 \quad \Rightarrow \quad x = 22/3.$$

5e Consolidate logarithms:

$$\log_2(x+1) + \log_2(x-1) = 3 \quad \Rightarrow \quad \log_2(x+1)(x-1) = 3 \quad \Rightarrow \quad 2^3 = x^2 - 1 \quad \Rightarrow \quad x = \pm 3.$$

But -3 is an extraneous solution (it results in the logarithm of a negative number in the original equation), so $x = 3$ is the only solution.

7 $\tan\varphi = 2$, $\sin\varphi = 2/\sqrt{5}$, $\sec\varphi = \sqrt{5}$, $\cos\varphi = 1/\sqrt{5}$, $\csc\varphi = \sqrt{5}/2$.

8 $67^\circ\, 55'\, 48''$

9 Let h be the height of the cloud. We have

$$\tan 72.35^\circ = \frac{h}{65} \quad \Rightarrow \quad h = 65(\tan 72.35^\circ) \approx 204.29.$$

The cloud is about 204 meters high.