Math 125 Exam #2 Key (Spring 2014)

1 Suppose that $f(x) = f(y)$. Then

$$\frac{x+2}{1-4x} = \frac{y+2}{1-4y} \;\Rightarrow\; (x+2)(1-4y) = (y+2)(1-4x)$$
$$\Rightarrow\; x - 4xy - 8y + 2 = y - 4xy - 8x + 2$$
$$\Rightarrow\; x - 8y = y - 8x \;\Rightarrow\; 9x = 9y \;\Rightarrow\; x = y.$$

Since $f(x) = f(y)$ implies that $x = y$, we conclude that f is one-to-one.

2 The fact that $g(-1) = (-1)^4 - 5 = -4 = 1^4 - 5 = g(1)$ shows g is not one-to-one.

3a Let $y = f(x)$, so by definition $x = f^{-1}(y)$. Now,

$$y = \frac{x+2}{1-4x} \;\Rightarrow\; y(1-4x) = x+2 \;\Rightarrow\; x(1+4y) = y-2 \;\Rightarrow\; x = \frac{y-2}{4y+1},$$

and therefore

$$f^{-1}(y) = \frac{y-2}{4y+1}.$$

3b We have

$$\text{Ran}(f^{-1}) = \text{Dom}(f) = (-\infty, \tfrac{1}{4}) \cup (\tfrac{1}{4}, \infty) \;\;\&\;\; \text{Ran}(f) = \text{Dom}(f^{-1}) = (-\infty, -\tfrac{1}{4}) \cup (-\tfrac{1}{4}, \infty).$$

4a $\left(3 + \sqrt{-16}\right) + \left(2 + \sqrt{-25}\right) = (3+4i) + (2+5i) = 5 + 9i$

4b We have

$$\frac{i}{2+i} \cdot \frac{2-i}{2-i} = \frac{2i - i^2}{4 - 2i + 2i - i^2} = \frac{1+2i}{5} = \frac{1}{5} + \frac{2}{5}i.$$

5a Factoring yields

$$9x(2+x) = 0 \;\Rightarrow\; 9x = 0 \;\text{ or }\; 2 + x = 0 \;\Rightarrow\; x = 0 \;\text{ or }\; x = -2,$$

so the solution set is $\{-2, 0\}$.

5b Complete the square:

$$x^2 + 6x = -13 \;\Rightarrow\; x^2 + 6x + 9 = -13 + 9 \;\Rightarrow\; (x+3)^2 = -4$$
$$\Rightarrow\; x + 3 = \pm\sqrt{-4} = \pm 2i \;\Rightarrow\; x = -3 \pm 2i,$$

so solution set is $\{-3 \pm 2i\}$.

6a Multiply by $x(x-6)$ to get $x - (x-6) = 6$, and then $6 = 6$. The equation is an identity, with solution set $(-\infty, \infty)$.

6b Cubing both sides gives $2x + 1 = -64$, and then $x = -65/2$. Solution set is $\{-65/2\}$.

6c Square both sides to get $7x + 4 = (x + 2)^2$, and thus $7x + 4 = x^2 + 4x + 4$. Rearranging: $x^2 - 3x = 0$. Factoring: $x(x - 3) = 0$. From this we obtain $x = 0, 3$, both of which are valid solutions. Solution set: $\{0, 3\}$.

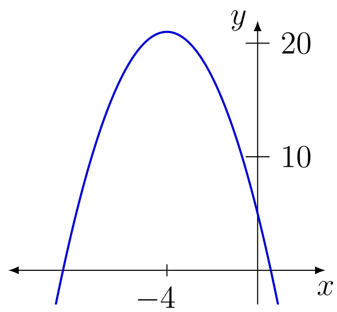
7a Write $f(x)$ in vertex form:

$$f(x) = -x^2 - 8x + 5 = -(x^2 + 8x + 16) + 5 + 16 = -(x + 4)^2 + 21.$$

So vertex is at $(-4, 21)$, and axis of symmetry is $x = -4$.

7b $f(x)$ has a maximum value, namely $f(-4) = 21$.

7c Graph is below.



8a We have

$$2x - 1 = 2 \quad \text{or} \quad 2x - 1 = -2.$$

Solving yields $x = 3/2$ or $x = -1/2$. Solution set: $\{-1/2, 3/2\}$.

8b We have

$$|x + 5| < 8 \quad \Rightarrow \quad -8 < x + 5 < 8 \quad \Rightarrow \quad -13 < x < 3.$$

Solution set: $(-13, 3)$.

8c $|6 - 4x| \geq 8$ implies that

$$6 - 4x \geq 8 \quad \text{or} \quad 6 - 4x \leq -8.$$

Solving these inequalities yields

$$x \leq -1/2 \quad \text{or} \quad x \geq 7/2,$$

and so the solution set is $(-\infty, -1/2] \cup [7/2, \infty)$.

9 We have

$$\begin{array}{r|rrrrr|r} -2 & 1 & 0 & 1 & 0 & -1 & 0 \\ & & -2 & 4 & -10 & 20 & -38 \\ \hline & 1 & -2 & 5 & -10 & 19 & -38 \end{array} \quad \longrightarrow \quad x^4 - 2x^3 + 5x^2 - 10x + 19 - \frac{38}{x + 2}.$$

10a The division

$$\begin{array}{r|rrrr|r} -2 & 1 & -7 & 9 & 27 & -54 \\ & & -2 & 18 & -54 & 54 \\ \hline & 1 & -9 & 27 & -27 & 0 \end{array}$$

followed by

$$\begin{array}{r|rrr|r} 3 & 1 & -9 & 27 & -27 \\ & & 3 & -18 & 27 \\ \hline & 1 & -6 & 9 & 0 \end{array}$$

shows that -2 and 3 are zeros for the function f, and we obtain the factorization

$$f(x) = (x+2)(x-3)(x^2 - 6x + 9) = (x+2)(x-3)^3$$

from the bottom row of numbers in the second division.

10b The solution set for $f(x) = 0$ is $\{-2, 3\}$.

11 In order to have rational coefficients and $2 - i$ as a zero, the Conjugate Zeros Theorem implies that $2 + i$ must also be a zero. So

$$f(x) = (x+1)[x - (2-i)][x - (2+i)] = (x+1)(x^2 - 4x + 5) = x^3 - 3x^2 + x + 5.$$