Math 125 Exam #4 Key (Spring 2013)

1a $(12.5^\circ)\left(\dfrac{\pi}{180^\circ}\right) = \dfrac{5}{72}\pi.$

1b $\dfrac{180^\circ}{\pi} \approx 57.30^\circ$

2 Amplitude is 4, period is 8π. If we write

$$4\sin\left[\frac{1}{4}\left(x-\left(-\frac{\pi}{2}\right)\right)\right]$$

we can see that the phase shift is $-\pi/2$.

3 We have

$$\frac{5\cos\varphi}{\sin^2\varphi}\cdot\frac{(\sin\varphi)(\sin\varphi-\cos\varphi)}{(\sin\varphi-\cos\varphi)(\sin\varphi+\cos\varphi)} \quad\Rightarrow\quad \frac{5\cos\varphi}{\sin^2\varphi+\cos\varphi\sin\varphi}$$

4 With a half-angle identity we obtain

$$\cos 15^\circ = \sqrt{\frac{1+\cos 30^\circ}{2}} = \sqrt{\frac{1+\sqrt{3}/2}{2}} = \sqrt{\frac{2+\sqrt{3}}{4}} = \frac{\sqrt{2+\sqrt{3}}}{2}.$$

5a $\dfrac{1+\cos^2 x}{\sin^2 x} = \dfrac{1+(1-\sin^2 x)}{\sin^2 x} = \dfrac{2}{\sin^2 x} - 1 = 2\csc^2 x - 1.$

5b We have

$$\begin{aligned}\frac{1+\sin x}{1-\sin x} &= \frac{1+\sin x}{1-\sin x}\cdot\frac{1+\sin x}{1+\sin x} = \frac{1+2\sin x+\sin^2 x}{1-\sin^2 x} = \frac{1+2\sin x+\sin^2 x}{\cos^2 x}\\ &= \sec^2 x + 2\tan x\sec x + \tan^2 x = (\sec x+\tan x)^2.\end{aligned}$$

6a Let $x = \tan^{-1}(\sqrt{3}/3)$, so $\tan x = \sqrt{3}/3$, which implies that $x = \pi/6$. Now,

$$\sin\left(\tan^{-1}\sqrt{3}/3\right) = \sin x = \sin\pi/6 = 1/2.$$

6b Let $x = \sin^{-1}(\sin 7\pi/6)$, so $x \in [-\pi/2, \pi/2]$ such that $\sin x = \sin 7\pi/6$. The only solution is $x = -\pi/6$.

7a Equation becomes $2\sin^2\theta + 7\sin\theta - 4 = 0$, and thus

$$(2\sin\theta - 1)(\sin\theta + 4) = 0.$$

Thus we must have $\sin\theta = -4$ or $\sin\theta = 1/2$. The first equation has no solution, but the second yields $\theta = \pi/6, 5\pi/6$. Solution set is $\{\pi/6, 5\pi/6\}$.

7b Using a trigonometric identity, equation becomes

$$(2\sin x\cos x)\cos x - \sin x = 0 \quad\Rightarrow\quad (\sin x)(2\cos^2 - 1) = 0,$$

Thus we must have $\sin x = 0$ or $\cos^2 x = 1/2$. The first equation yields $x = 0, \pi$, and the second equation becomes $\cos x = \pm 1/\sqrt{2}$, which yields $x = \pi/4, 3\pi/4, 5\pi/4, 7\pi/4$. Solution set is

$$\{0, \pi, \pi/4, 3\pi/4, 5\pi/4, 7\pi/4\}.$$

7c We have

$$(1 + \tan^2 t) - 2\tan^2 t = 0 \quad\Rightarrow\quad 1 - \tan^2 t = 0 \quad\Rightarrow\quad \tan t = \pm 1.$$

Solution set is $\{\pi/4, 3\pi/4, 5\pi/4, 7\pi/4\}$.

8a First, $A = 180° - 10° - 100° = 70°$. Now, $\dfrac{c}{\sin 100°} = \dfrac{2}{\sin 10°} \quad\Rightarrow\quad c = 11.343$, and $\dfrac{a}{\sin 70°} = \dfrac{2}{\sin 10°} \quad\Rightarrow\quad a = 10.823$.

8b By the Law of Sines we have

$$\frac{\sin B}{b} = \frac{\sin A}{a} \quad\Rightarrow\quad \sin B = \frac{b\sin A}{a} = \frac{18.4\sin 89°}{15.6} \approx 1.18,$$

which has no solution since $\sin B$ cannot be greater than 1.

8c We have

$$\begin{aligned}\frac{\sin B}{10} = \frac{\sin 10°}{3} \quad &\Rightarrow\quad \sin B = 0.57883\\ &\Rightarrow\quad \sin^{-1}(\sin B) = \sin^{-1}(0.57883) = 35.366°\\ &\Rightarrow\quad \sin B = \sin 35.366°.\end{aligned}$$

One solution to this equation is of course $B_1 = 35.366°$; however B could also be the Quadrant II angle

$$B_2 = 180° - \sin^{-1}(0.57883) = 144.63°$$

(see very pretty picture below).

For the angle B_1 we get $C_1 = 134.634°$, and then by the Law of Cosines we obtain

$$c_1^2 = a^2 + b^2 - 2ab\cos C_1 = 3^2 + 10^2 - 2(3)(10)\cos 134.634° = 151.156 \quad\Rightarrow\quad c_1 = 12.29.$$

So one possible triangle (rounding to the nearest hundredth) has

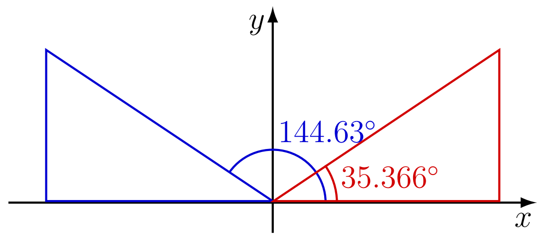
$$B_1 = 35.37°, \quad C_1 = 134.63°, \quad c_1 = 12.29.$$

For the angle B_2 we get $C_2 = 25.370°$, and then by the Law of Cosines we obtain

$$c_2^2 = a^2 + b^2 - 2ab\cos C_2 = 3^2 + 10^2 - 2(3)(10)\cos 25.370° = 54.786 \quad \Rightarrow \quad c_1 = 7.40.$$

So another possible triangle has

$$B_2 = 144.63°, \quad C_2 = 25.37°, \quad c_2 = 7.40.$$



8d The Law of Cosines is necessary here:

$$\begin{aligned} c^2 = a^2 + b^2 - 2ab\cos C \quad &\Rightarrow \quad 6^2 = 4^2 + 3^2 - 2(4)(3)\cos C \\ &\Rightarrow \quad \cos C = -11/24 \\ &\Rightarrow \quad C = \cos^{-1}(-11/24) = 117.28°. \end{aligned}$$

And

$$\begin{aligned} b^2 = a^2 + c^2 - 2ac\cos B \quad &\Rightarrow \quad 3^2 = 4^2 + 6^2 - 2(4)(6)\cos B \\ &\Rightarrow \quad \cos B = 43/48 \\ &\Rightarrow \quad B = \cos^{-1}(43/48) = 26.38°. \end{aligned}$$

Finally, $A = 180° - 26.38° - 117.28° = 36.34°$.