Math 125 Exam #2 Key (Spring 2013)

1 Suppose that $f(x) = f(y)$. Then $2x^3 - 5 = 2y^3 - 5$, and

$$2x^3 - 5 = 2y^3 - 5 \quad\Rightarrow\quad 2x^3 = 2y^3 \quad\Rightarrow\quad x^3 = y^3 \quad\Rightarrow\quad x = y.$$

Since $f(x) = f(y)$ implies that $x = y$, we conclude that f is one-to-one.

2 The fact that $g(-1) = (-1)^4 - 3(-1)^2 = -2 = 1^4 - 3(1)^2 = g(1)$ shows that g is not one-to-one.

3 Let $y = h(x)$, so by definition $x = h^{-1}(y)$. Now,

$$y = \frac{5x-3}{2x+1} \quad\Rightarrow\quad (2x+1)y = 5x - 3 \quad\Rightarrow\quad x(5-2y) = y + 3 \quad\Rightarrow\quad x = \frac{y+3}{5-2y},$$

and therefore

$$h^{-1}(y) = \frac{y+3}{5-2y}.$$

4a Let $y = r(x)$, so by definition $x = r^{-1}(y)$. Now,

$$y = \frac{x-1}{x+2} \quad\Rightarrow\quad xy + 2y = x - 1 \quad\Rightarrow\quad x(y-1) = -2y - 1 \quad\Rightarrow\quad x = \frac{2y+1}{1-y},$$

and therefore

$$r^{-1}(y) = \frac{2y+1}{1-y}.$$

4b $\mathrm{Ran}(r^{-1}) = \mathrm{Dom}(r) = (-\infty, -2) \cup (-2, \infty)$ and $\mathrm{Ran}(r) = \mathrm{Dom}(r^{-1}) = (-\infty, 1) \cup (1, \infty)$.

5a $(4 - 3i)(2 + 9i) = 35 + 30i$

5b We have

$$\frac{3-2i}{2-i} \cdot \frac{2+i}{2+i} = \frac{6 + 3i - 4i - 2i^2}{4 + 2i - 2i - i^2} = \frac{6 - i + 2}{4+1} = \frac{8}{5} - \frac{1}{5}i.$$

6a Factoring yields

$$9x(2+x) = 0 \quad\Rightarrow\quad 9x = 0 \quad\text{or}\quad 2 + x = 0 \quad\Rightarrow\quad x = 0 \quad\text{or}\quad x = -2,$$

so the solution set is $\{-2, 0\}$.

6b Complete the square:

$$\begin{aligned} x^2 + 6x = -13 \quad&\Rightarrow\quad x^2 + 6x + 9 = -13 + 9 \quad\Rightarrow\quad (x+3)^2 = -4 \\ &\Rightarrow\quad x + 3 = \pm\sqrt{-4} = \pm 2i \quad\Rightarrow\quad x = -3 \pm 2i, \end{aligned}$$

so solution set is $\{-3 \pm 2i\}$.

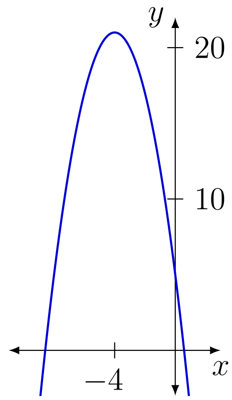
7a Write $f(x)$ in vertex form:

$$f(x) = -x^2 - 8x + 5 = -(x^2 + 8x + 16) + 5 + 16 = -(x+4)^2 + 21.$$

So vertex is at $(-4, 21)$, and axis of symmetry is $x = -4$.

7b $f(x)$ has a maximum value, namely $f(-4) = 21$.

7c Graph is below.



8a We have

$$2x - 1 = 2 \quad \text{or} \quad 2x - 1 = -2.$$

Solving yields $x = 3/2$ or $x = -1/2$. Solution set: $\{-1/2, 3/2\}$.

8b We have

$$|x + 5| < 8 \quad \Rightarrow \quad -8 < x + 5 < 8 \quad \Rightarrow \quad -13 < x < 3.$$

Solution set: $(-13, 3)$.

8c $|6 - 4x| \geq 8$ implies that

$$6 - 4x \geq 8 \quad \text{or} \quad 6 - 4x \leq -8.$$

Solving these inequalities yields

$$x \leq -1/2 \quad \text{or} \quad x \geq 7/2,$$

and so the solution set is $(-\infty, -1/2] \cup [7/2, \infty)$.

9 We have

$$\begin{array}{r|rrrrr|r} 3 & 1 & 0 & 1 & 0 & -1 & 0 \\ & & 3 & 9 & 30 & 90 & 267 \\ \hline & 1 & 3 & 10 & 30 & 89 & 267 \end{array} \quad \longrightarrow \quad x^4 + 3x^3 + 10x^2 + 30x + 89 + \frac{267}{x-3}.$$

10 The division

$$
\begin{array}{r|rrrr|r}
-2 & 1 & -7 & 9 & 27 & -54 \\
 & & -2 & 18 & -54 & 54 \\
\hline
 & 1 & -9 & 27 & -27 & 0
\end{array}
$$

followed by

$$
\begin{array}{r|rrr|r}
3 & 1 & -9 & 27 & -27 \\
 & & 3 & -18 & 27 \\
\hline
 & 1 & -6 & 9 & 0
\end{array}
$$

shows that -2 and 3 are zeros for the function f, and we obtain the factorization

$$f(x) = (x+2)(x-3)(x^2-6x+9) = (x+2)(x-3)^3$$

from the bottom row of numbers in the second division.

11 By the Factor Theorem we obtain

$$f(x) = x(2x+1)^2(x-1)^2 = 4x^5 - 4x^4 - 3x^3 + 2x^2 + x.$$

12 In order to have rational coefficients and $2-i$ as a zero, the Conjugate Zeros Theorem implies that $2+i$ must also be a zero. So

$$f(x) = (x+1)[x-(2-i)][x-(2+i)] = (x+1)(x^2-4x+5) = x^3 - 3x^2 + x + 5.$$

13 The division

$$
\begin{array}{r|rrrr|r}
1 & 1 & 5 & -27 & 31 & -10 \\
 & & 1 & 6 & -21 & 10 \\
\hline
 & 1 & 6 & -21 & 10 & 0
\end{array}
$$

followed by

$$
\begin{array}{r|rrr|r}
2 & 1 & 6 & -21 & 10 \\
 & & 2 & 16 & -10 \\
\hline
 & 1 & 8 & -5 & 0
\end{array}
$$

shows that 1 and 2 are zeros for the function f, and we obtain the factorization

$$f(x) = (x-1)(x-2)(x^2+8x-5)$$

from the bottom row of numbers in the second division. The other zeros for f are the solutions to the quadratic equation

$$x^2 + 8x - 5 = 0,$$

which by the quadratic formula are found to be $-4 \pm \sqrt{21}$. That is, the zeros of f are 1, 2, $-4 \pm \sqrt{21}$.