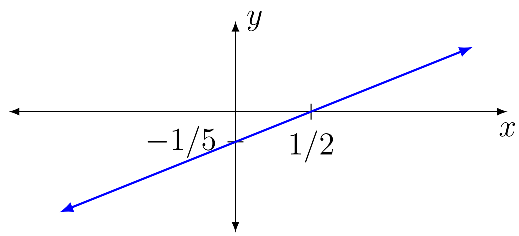
Math 125 Exam #1 Key (Spring 2013)

1a $\sqrt{(-2-(-6))^2+(5-(-1))^2} = \sqrt{4^2+6^2} = \sqrt{52} = 2\sqrt{13}$

1b $\left(\dfrac{-2+(-6)}{2}, \dfrac{5+(-1)}{2}\right) = (-4,2)$

2 Center is $(-4,2)$ and radius is $3/2$, so equation is $(x+4)^2+(y-2)^2 = 9/4$.

3 Writing the equation as $y = \frac{2}{5}x - \frac{1}{5}$ makes it clear that slope is $m = \frac{2}{5}$. Intercepts are $\left(0, -\frac{1}{5}\right)$ and $\left(\frac{1}{2}, 0\right)$.



(Actually the original problem was $2x - 5x = 1$, which though a typo is nevertheless a valid problem encountered in homework so it's fine if anyone worked it out as given. In this case the equation can be written as simply $x = -1/3$, which has undefined slope, an x-intercept at $(-1/3, 0)$, and a graph that is just a vertical line containing the point $(-1/3, 0)$. There is no y-intercept.)

4 We have

$$\begin{aligned} f(0) &= -0^2 + 3(0) - 2 = -2, \\ f(4) &= -4^2 + 3(4) - 2 = -6, \\ f(-x) &= -(-x)^2 + 3(-x) - 2 = -x^2 - 3x - 2. \end{aligned}$$

5a We have

$$\mathrm{Dom}(f) = \{x : x+7 \neq 0\} = \{x : x \neq -7\} = (-\infty, -7) \cup (-7, \infty)$$

5b We have

$$\mathrm{Dom}(g) = \{x : 4-5x \geq 0\} = \{x : 5x \leq 4\} = \{x : x \leq 4/5\} = (-\infty, 4/5]$$

5c We have

$$\mathrm{Dom}(h) = \{x : x-8 \neq 0 \ \&\ x+9 \geq 0\} = \{x : x \neq 8 \ \&\ x \geq -9\} = [-9, 8) \cup (8, \infty)$$

6a Finding fg:

$$(fg)(x) = f(x)g(x) = \frac{2x-5}{x+7} \cdot \sqrt{4-5x} = \frac{(2x-5)\sqrt{4-5x}}{x+7}.$$

Now for the domain:

$$\begin{aligned}\mathrm{Dom}(fg) &= \mathrm{Dom}(f) \cap \mathrm{Dom}(g) = \big((-\infty,-7) \cup (-7,\infty)\big) \cap (-\infty, 4/5] \\ &= (-\infty,-7) \cup (-7, 4/5].\end{aligned}$$

6b Finding h/f:

$$(h/f)(x) = h(x)/f(x) = \frac{\sqrt{x+9}}{x-8} \div \frac{2x-5}{x+7} = \frac{(x+7)\sqrt{x+9}}{(x-8)(2x-5)}.$$

Now for the domain:

$$\begin{aligned}\mathrm{Dom}(h/f) &= \{x : x \in \mathrm{Dom}(f) \cap \mathrm{Dom}(h) \ \&\ f(x) \neq 0\} \\ &= \big\{x : x \in \big((-\infty,-7) \cup (-7,\infty)\big) \cap \big([-9,8) \cup (8,\infty)\big) \ \&\ f(x) \neq 0\big\} \\ &= \{x : x \in [-9,-7) \cup (-7,8) \cup (8,\infty) \ \&\ x \neq 5/2\} \\ &= [-9,-7) \cup (-7,5/2) \cup (5/2,8) \cup (8,\infty).\end{aligned}$$

6c Finding $g \circ g$:

$$(g \circ g)(x) = g(g(x)) = g\big(\sqrt{4-5x}\big) = \sqrt{4-5\sqrt{4-5x}}.$$

Now for the domain:

$$\begin{aligned}\mathrm{Dom}(g \circ g) &= \{x : x \in \mathrm{Dom}(g) \ \&\ g(x) \in \mathrm{Dom}(g)\} \\ &= \big\{x : x \in (-\infty, 4/5] \ \&\ \sqrt{4-5x} \in (-\infty, 4/5]\big\},\end{aligned}$$

where

$$\sqrt{4-5x} \in (-\infty, 4/5] \quad \Leftrightarrow \quad \sqrt{4-5x} \leq 4/5 \quad \Leftrightarrow \quad 0 \leq 4-5x \leq 16/25 \quad \Leftrightarrow \quad \tfrac{84}{125} \leq x \leq \tfrac{4}{5},$$

and so

$$\mathrm{Dom}(g \circ g) = \big\{x : x \in \big(-\infty, \tfrac{4}{5}\big] \ \&\ x \in \big[\tfrac{84}{125}, \tfrac{4}{5}\big]\big\} = \big[\tfrac{84}{125}, \tfrac{4}{5}\big].$$

7a We have

$$(F \circ G)(x) = F(x^2-5) = \sqrt{(x^2-5)+5} = \sqrt{x^2} = |x|,$$

with

$$\begin{aligned}\mathrm{Dom}(F \circ G) &= \{x : x \in \mathrm{Dom}(G) \ \&\ G(x) \in \mathrm{Dom}(F)\} \\ &= \{x : x \in (-\infty,\infty) \ \&\ x^2 - 5 \in [-5,\infty)\} \\ &= \{x : x^2 - 5 \geq -5\} = \{x : x^2 \geq 0\} = (-\infty,\infty).\end{aligned}$$

7b We have

$$(G \circ F)(x) = G\big(\sqrt{x+5}\,\big) = \big(\sqrt{x+5}\,\big)^2 - 5 = (x+5) - 5 = x,$$

with

$$\begin{aligned} \mathrm{Dom}(G \circ F) &= \{x : x \in \mathrm{Dom}(F) \ \&\ F(x) \in \mathrm{Dom}(G)\} \\ &= \{x : x \in [-5, \infty) \ \&\ \sqrt{x+5} \in (-\infty, \infty)\} \\ &= \{x : x \in [-5, \infty) \ \&\ x \in [-5, \infty)\} = [-5, \infty). \end{aligned}$$

8 Let $g(x) = \sqrt{3x+7}$ and $f(x) = 1/x$. Another possibility: $g(x) = 3x+7$ and $f(x) = 1/\sqrt{x}$.

9 $\varphi(-5) = 1$, $\varphi(0) = 1$, $\varphi(1) = 3$, $\varphi(3) = 5$.

10a $\mathrm{Dom}(H) = (-\infty, \infty)$ and $\mathrm{Ran}(H) = (-\infty, -2] \cup [3, \infty)$.

10b H is a piecewise-defined function as follows:

$$H(x) = \begin{cases} x, & \text{if } x \le -2 \\ 3, & \text{if } -2 < x \le 3 \\ \frac{3}{2}x - \frac{3}{2}, & \text{if } x \ge 3 \end{cases}$$

11 Substituting $-x$ for x yields $y^3 = 2(-x)^2$, which is equivalent to the original equation $y^3 = 2x^2$ and so there is symmetry about the y-axis.

Substituting $-y$ for y yields $-y^3 = 2x^2$, which is *not* equivalent to the original equation (i.e. it has a different solution set). No symmetry about the x-axis.

Substituting $-x$ for x and $-y$ for y yields $-y^3 = 2x^2$, again not equivalent to the original equation. No symmetry about the origin.

12 Call the function f, so that $f(x) = \sqrt{x+5} - 3$.