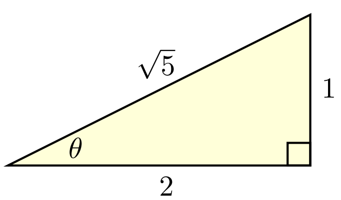
Math 125 Exam #4 Key (Spring 2011)

1. Let $\theta = \sin^{-1}(\sin 11\pi/8)$. Then θ is a value in $[-\pi/2, \pi/2]$ such that $\sin\theta = \sin(11\pi/8)$, and thus $\theta = -3\pi/8$. That is, $\sin^{-1}(\sin 11\pi/8) = -3\pi/8$.

2. As in the definition for the inverse sine function, we restrict f to $[-\pi/2, \pi/2]$ to obtain a function that is one-to-one and therefore has an inverse, so $\text{Dom}(f) = [-\pi/2, \pi/2] = \text{Ran}(f^{-1})$. As for the range of f, we have $-1 \le \sin x \le 1 \;\Rightarrow\; -7 \le -7\sin x \le 7 \;\Rightarrow\; -2 \le 5 - 7\sin x \le 12$, so $\text{Ran}(f) = [-2, 12] = \text{Dom}(f^{-1})$. Finally, let $y = f(x)$, so $y = 5 - 7\sin x$ and we get $x = \sin^{-1}\left(\dfrac{y-5}{-7}\right)$. Since $y = f(x)$ implies that $x = f^{-1}(y)$, the result is $f^{-1}(y) = \sin^{-1}\left(\dfrac{5-y}{7}\right)$.

3a. Let $\theta = \sin^{-1}(-1/2)$, so $\sin\theta = -1/2$ for $\theta \in [-\pi/2, \pi/2]$, which implies that $\theta = -\pi/6$. Now, $\tan(\sin^{-1}(-1/2)) = \tan(-\pi/6) = -1/\sqrt{3}$.

3b. Let $\theta = \tan^{-1}(1/2)$, so $\tan\theta = 1/2$ for $\theta \in (-\pi/2, \pi/2)$, which implies that θ is the angle in the right triangle shown below. Then $\sec(\tan^{-1}(1/2)) = \sec\theta = \sqrt{5}/2$.



4. $\theta = \csc^{-1}(12) \;\Rightarrow\; \csc\theta = 12 \;\Rightarrow\; \sin\theta = 1/12$ for $\theta \in [-\pi/2, 0) \cup (0, \pi/2]$ (which is the range of the $\csc^{-1}$ function). Using a calculator, we find that $\theta = \sin^{-1}(1/12) \approx 0.083 \approx 0.08$.

5a. $\dfrac{\sec\theta}{\csc\theta} + \dfrac{\sin\theta}{\cos\theta} = \dfrac{\sin\theta}{\cos\theta} + \dfrac{\sin\theta}{\cos\theta} = 2 \cdot \dfrac{\sin\theta}{\cos\theta} = 2\tan\theta$.

5b. $3\sin^2\theta + 4\cos^2\theta = 3(1 - \cos^2\theta) + 4\cos^2\theta = 3 - 3\cos^2\theta + 4\cos^2\theta = 3 + \cos^2\theta$.

5c. $\cos\left(\dfrac{3\pi}{2} + \theta\right) = \cos(3\pi/2)\cos\theta - \sin(3\pi/2)\sin\theta = 0 \cdot \cos\theta - (-1)\sin\theta = \sin\theta$.

5d. $\cos^4\theta - \sin^4\theta = (\cos^2\theta - \sin^2\theta)(\cos^2\theta + \sin^2\theta) = \cos(2\theta) \cdot 1 = \cos(2\theta)$.

6a. Using $\tan\left(\dfrac{\theta}{2}\right) = \dfrac{1-\cos\theta}{\sin\theta}$, we get $\tan\left(\dfrac{17\pi}{12}\right) = \dfrac{1-\cos(17\pi/6)}{\sin(17\pi/6)} = \dfrac{1-\cos(5\pi/6)}{\sin(5\pi/6)} = \dfrac{1-(-\sqrt{3}/2)}{1/2} = 2 + \sqrt{3}$.

6b. $\cos 40^\circ \cos 10^\circ + \sin 40^\circ \sin 10^\circ = \cos(40^\circ - 10^\circ) = \cos 30^\circ = \sqrt{3}/2$.

6c. The angle 165° is in Quadrant II where cosine is negative, so $\cos 165^\circ = -\sqrt{\dfrac{1+\cos 330^\circ}{2}} = -\sqrt{\dfrac{1+\sqrt{3}/2}{2}}$ $= -\sqrt{\dfrac{2+\sqrt{3}}{4}} = -\dfrac{\sqrt{6}+\sqrt{2}}{4}$, where the last equality holds since $\left(\dfrac{\sqrt{6}+\sqrt{2}}{4}\right)^2 = \dfrac{6+2\sqrt{12}+2}{16} = \dfrac{8+4\sqrt{3}}{16}$

$= \dfrac{2+\sqrt{3}}{4}$.

7a. $3\sqrt{2}\cos\theta = -3 \;\Rightarrow\; \cos\theta = -1/\sqrt{2} \;\Rightarrow\; \theta = 3\pi/4, 5\pi/4$.

7b. $2\sin^2\theta = 3(1-\cos\theta) \;\Rightarrow\; 2(1-\cos^2\theta) = 3-3\cos\theta \;\Rightarrow\; 2\cos^2\theta - 3\cos\theta + 1 = 0 \;\Rightarrow\; (2\cos\theta-1)(\cos\theta-1) = 0$, so either $\cos\theta = 1/2$ or $\cos\theta = 1$. Solving these equations gives $\theta = \pi/3, 5\pi/3, 0$.

7c. $\cos(2\theta) = 2 - 2\sin^2\theta \;\Rightarrow\; \cos^2\theta - \sin^2\theta = 2 - 2\sin^2\theta \;\Rightarrow\; (1-\sin^2\theta) - \sin^2\theta = 2 - 2\sin^2\theta \;\Rightarrow\; 1 - 2\sin^2\theta = 2 - 2\sin^2\theta \;\Rightarrow\; 1 = 2$. No solution.

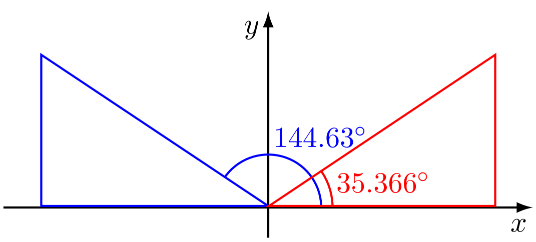
8. First, $C = 180° - 28° - 74° = 78°$. Now, by Ye Olde Law of Sines, $\dfrac{a}{\sin 74°} = \dfrac{9}{\sin 78°} \;\Rightarrow\; a = 8.845$, and $\dfrac{b}{\sin 28°} = \dfrac{9}{\sin 78°} \;\Rightarrow\; b = 4.320$.

9a. First, $A = 180° - 10° - 100° = 70°$. Now, $\dfrac{c}{\sin 100°} = \dfrac{2}{\sin 10°} \;\Rightarrow\; c = 11.343$, and $\dfrac{a}{\sin 70°} = \dfrac{2}{\sin 10°} \;\Rightarrow\; a = 10.823$.

9b. $\dfrac{\sin B}{10} = \dfrac{\sin 10°}{3} \;\Rightarrow\; \sin B = 0.57883 \;\Rightarrow\; \sin^{-1}(\sin B) = \sin^{-1}(0.57883) = 35.366° \;\Rightarrow\; \sin B = \sin 35.366°$. One solution to this equation is of course $B_1 = 35.366°$; however B could also be the Quadrant II angle $B_2 = 180° - \sin^{-1}(0.57883) = 144.63°$ (see very pretty picture below).

For the angle B_1 we get $C_1 = 134.634°$, and then by the Law of Cosines we obtain $c_1^2 = a^2 + b^2 - 2ab\cos C_1 = 3^2 + 10^2 - 2(3)(10)\cos 134.634° = 151.156 \;\Rightarrow\; c_1 = 12.29$. So one possible triangle (rounding to the nearest hundredth) has $B_1 = 35.37°$, $C_1 = 134.63°$, $c_1 = 12.29$.

For the angle B_2 we get $C_2 = 25.370°$, and then by the Law of Cosines we obtain $c_2^2 = a^2 + b^2 - 2ab\cos C_2 = 3^2 + 10^2 - 2(3)(10)\cos 25.370° = 54.786 \;\Rightarrow\; c_1 = 7.40$. So another possible triangle has $B_2 = 144.63°$, $C_2 = 25.37°$, $c_2 = 7.40$.



9c. The Law of Cosines is necessary here: $c^2 = a^2 + b^2 - 2ab\cos C \;\Rightarrow\; 6^2 = 4^2 + 3^2 - 2(4)(3)\cos C \;\Rightarrow\; \cos C = -11/24 \;\Rightarrow\; C = \cos^{-1}(-11/24) = 117.28°$. And $b^2 = a^2 + c^2 - 2ac\cos B \;\Rightarrow\; 3^2 = 4^2 + 6^2 - 2(4)(6)\cos B \;\Rightarrow\; \cos B = 43/48 \;\Rightarrow\; B = \cos^{-1}(43/48) = 26.38°$. Finally, $A = 180° - 26.38° - 117.28° = 36.34°$.