Math 125 Exam #3 Key (Spring 2011)

1. $8^{3-4x} = 64^{x-1} \;\Rightarrow\; 2^{3(3-4x)} = 2^{6(x-1)} \;\Rightarrow\; 3(3-4x) = 6(x-1) \;\Rightarrow\; 9 - 12x = 6x - 6 \;\Rightarrow\; 18x = 15 \;\Rightarrow\; x = 5/6$

2. $\text{Dom}(f) = \{x \mid \ln(x-1) \geq 0\} = \{x \mid x - 1 \geq e^0\} = \{x \mid x \geq 2\} = [2, \infty)$.

3. Let $f(x) = y$, so $y = 8 - \log_5(3x-2)$, and then $\log_5(3x-2) = 8 - y$ gives $5^{8-y} = 3x - 2$, and finally $x = \dfrac{5^{8-y}+2}{3}$. Since $f^{-1}(y) = x$, we obtain $f^{-1}(y) = \dfrac{5^{8-y}+2}{3}$.

4a. $\log_3(2-7x) = 2 \;\Rightarrow\; 3^2 = 2 - 7x \;\Rightarrow\; 7x = -7 \;\Rightarrow\; x = -1$.

4b. Since $\log_5$ is a one-to-one function we obtain $2x + 3 = 3$, and thus $x = 0$.

4c. Write as $\log_8(x+6) + \log_8(x+4) = 1$, which gives $\log_8[(x+6)(x+4)] = 1 \;\Rightarrow\; (x+6)(x+4) = 8 \;\Rightarrow\; x^2 + 10x + 16 = 0 \;\Rightarrow\; (x+8)(x+2) = 0 \;\Rightarrow\; x = -8, -2$. However, $x = -8$ results in the logarithm of a negative number in the original equation, so it is extraneous. Solution: $x = -2$.

4d. $7^x = 50 \;\Rightarrow\; \ln(7^x) = \ln(50) \;\Rightarrow\; x \cdot \ln(7) = \ln(50) \;\Rightarrow\; x = \dfrac{\ln(50)}{\ln(7)} \approx 2.010$.

5. $\log_4(x^2-1) - 2\log_4(x+1) = \log_4(x^2-1) - \log_4(x+1)^2 = \log_4\left[\dfrac{x^2-1}{(x+1)^2}\right] = \log_4\left(\dfrac{x-1}{x+1}\right)$

6. $37.419° = 37° + (0.419°)\left(\dfrac{60'}{1°}\right) = 37° + 25.14' = 37° + 25' + (0.14')\left(\dfrac{60''}{1'}\right) = 37° + 25' + 8.4'' \approx 37°25'8''$

7a. $-135° \cdot \dfrac{\pi}{180°} = -\dfrac{135}{180}\pi = -\dfrac{3}{4}\pi$ (note that radian measure is technically unitless).

7b. $\dfrac{5\pi}{12} \cdot \dfrac{180°}{\pi} = \dfrac{5}{12} \cdot 180° = 75°$.

8. A sector of a circle with radius 30 feet is being covered. The full area of such a circle is $A = \pi r^2 = \pi \cdot 30^2 = 900\pi \text{ ft}^2$, and the fraction of the circle being covered by the sprinkler is $135/360 = 3/8$. Thus, the area of lawn receiving water is $\frac{3}{8} \cdot 900\pi \text{ ft}^2$, or $\frac{675}{2}\pi \text{ ft}^2$ (approximately 1060 square feet). Notice the formula $A = \frac{1}{2}\theta r^2$ gives the same result, but remembering the formula isn't necessary if you employ a little reasoning.

9. $\sin\theta = -\frac{12}{13}$, $\cos\theta = \frac{5}{13}$, $\tan\theta = -\frac{12}{5}$, $\sec\theta = \frac{13}{5}$, $\csc\theta = -\frac{13}{12}$, $\cot\theta = -\frac{5}{12}$.

10. The angle 540° has the same terminal side as 180°, so $\sec 540° = \sec 180° = 1/\cos 180° = 1/-1 = -1$.

11. We must be in Quadrant II with $x = -3$, $y = 4$ and $r = 5$. Hence $\tan\theta = -\frac{4}{3}$, $\cot\theta = -\frac{3}{4}$, $\sec\theta = -\frac{5}{3}$, $\csc\theta = \frac{5}{4}$.

12. We must have $x = -4$, $y = -3$ and $r = 5$. Hence $\sin\theta = -\frac{3}{5}$, $\tan\theta = \frac{3}{4}$, $\cot\theta = \frac{4}{3}$, $\sec\theta = -\frac{5}{4}$, $\csc\theta = -\frac{5}{3}$.

13. Period is $\dfrac{2\pi}{3/2} = \dfrac{4}{3}\pi$ and amplitude is $\left|-\dfrac{1}{2}\right| = \dfrac{1}{2}$. The cosine function has a period of 2π, so really it comes down to figuring out what p must be so that $\frac{3}{2}(x+p) = \frac{3}{2}x + 2\pi$; solving this gives $\frac{3}{2}p = 2\pi$ and finally $p = \frac{4}{3}\pi$. Now, letting $f(x) = -\frac{1}{2}\cos(\frac{3}{2}x)$, we see

$$f\left(x + \frac{4}{3}\pi\right) = -\frac{1}{2}\cos\left(\frac{3}{2}\left(x + \frac{4}{3}\pi\right)\right) = -\frac{1}{2}\cos\left(\frac{3}{2}x + 2\pi\right) = -\frac{1}{2}\cos\left(\frac{3}{2}x\right) = f(x),$$

which shows that f has period $\frac{4}{3}\pi$. Once again a formula isn't necessary if a little reasoning is used, though it is crucial that one remembers the definition for the period of a function.