Math 125 Exam #1 Key (Spring 2011)

1. $D = \sqrt{(12-2)^2 + (3-(-5))^2} = \sqrt{164} = 2\sqrt{41}$.

2. For y-intercept: set $x = 0$ and solve for y to get $y = 9$; so $(0, 9)$ is the y-intercept. For x-intercept(s): set $y = 0$ to get $9x^2 = 36 \;\Rightarrow\; x = \pm 2$; so $(\pm 2, 0)$ are the x-intercepts.

3. $4(x-4) - 5 \cdot 4(x+1) = 21(x+1)(x-4) \;\Rightarrow\; 21x^2 - 47x - 48 = 0 \;\Rightarrow\; x = \dfrac{47 \pm \sqrt{47^2 - 4(21)(-48)}}{42} = \dfrac{47 \pm 79}{42} \in \left\{3,\ -\dfrac{16}{21}\right\}$.

4. Slope of line is $m = \dfrac{-8-(-5)}{-6-2} = \dfrac{3}{8}$. Now use the point-slope formula to get $y + 5 = \dfrac{3}{8}(x-2)$, which leads to $y = \dfrac{3}{8}x - \dfrac{23}{4}$.

5. Complete squares: $(x^2 - x) + (y^2 + 2y) = -1 \;\Rightarrow\; \left(x^2 - x + \left(-\frac{1}{2}\right)^2\right) + \left(y^2 + 2y + 1^2\right) = -1 + \left(-\frac{1}{2}\right)^2 + 1^2 \;\Rightarrow\; \left(x - \frac{1}{2}\right)^2 + (y+1)^2 = \frac{1}{4}$. From this it can be seen that center of the circle is at $\left(\frac{1}{2}, -1\right)$ and radius is $\frac{1}{2}$.

6. $f(-1) = -\frac{1}{2}$ and $f(c-1) = \dfrac{c-1}{(c-1)^2 + 1} = \dfrac{c-1}{c^2 - 2c + 2}$.

7a. $\text{Dom}(f) = \{x \mid x \neq \pm 6\} = (-\infty, -6) \cup (-6, 6) \cup (6, \infty)$.

7b. $\text{Dom}(g) = \{x \mid 3x \geq 12\} = \{x \mid x \geq 4\} = [4, \infty)$.

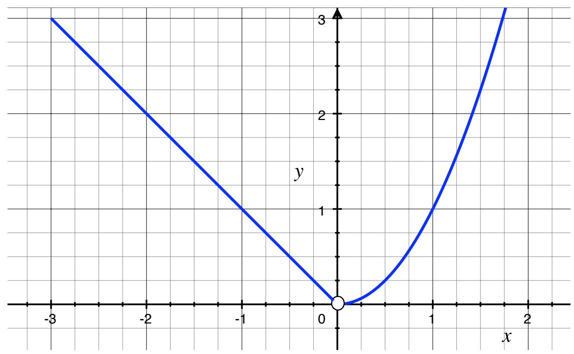
7c. $\text{Dom}(f+g) = \text{Dom}(f) \cap \text{Dom}(g) = [4, 6) \cup (6, \infty)$.

8a. $f(-2) = 2(-2)^2 - (-2) - 1 = 9$, and the corresponding point on the graph is $(-2, 9)$.

8b. We have $2x^2 - x - 1 = -1$, whence $2x^2 - x = 0 \;\Rightarrow\; x(2x-1) = 0 \;\Rightarrow\; x = 0, \frac{1}{2}$, and the corresponding points on the graph are $(0, -1)$ and $(1/2, -1)$.

9a. $\text{Dom}(f) = [-3, 0) \cup (0, \infty)$ and $\text{Ran}(f) = (0, \infty)$.

9b. I had better luck getting the graph to appear just as it's supposed to this semester. Progress!



10. Start with $f(x) = \sqrt{x}$. Shift down 4 units gives a new function: $g(x) = f(x) - 4$. Reflecting g about the x-axis gives a new function: $h(x) = -g(x)$. Shifting right by 7 units gives yet another new function: $k(x) = h(x-7)$. Our result is: $k(x) = h(x-7) = -g(x-7) = -[f(x-7)-4]$, or $y = -(\sqrt{x-7}-4) = -\sqrt{x-7}+4$.

11. In general $A = \pi r^2$. But circumference C is given by $C = 2\pi r$ and we know that C must equal x. Hence $x = 2\pi r$, which yields $r = \dfrac{x}{2\pi}$. Therefore A as a function of x is given by: $A(x) = \pi \left(\dfrac{x}{2\pi}\right)^2$, or $A(x) = \dfrac{x^2}{4\pi}$.

12. $(f \circ g)(4) = f(g(4)) = f(12) = \sqrt{13}$ and $(g \circ f)(2) = g(f(2)) = g(\sqrt{3}) = 3\sqrt{3}$.

13a $(f \circ f)(x) = f(f(x)) = \dfrac{f(x)-5}{f(x)+1} = \dfrac{\dfrac{x-5}{x+1} - 5}{\dfrac{x-5}{x+1} + 1} = \dfrac{2x+5}{2-x}$.

13b. First, Dom $f = \{x \mid x \neq -1\}$ and Dom $g = \{x \mid x \neq 3\}$. Now, by definition, Dom $g \circ f = \{x \mid x \in \text{Dom } f \text{ and } f(x) \in \text{Dom } g\} = \left\{x \,\middle|\, x \neq -1 \text{ and } \dfrac{x-5}{x+1} \neq 3\right\} = \{x \mid x \neq -1 \text{ and } x \neq -4\}$.

13c. By definition, Dom $f \circ g = \{x \mid x \in \text{Dom } g \text{ and } g(x) \in \text{Dom } f\} = \left\{x \,\middle|\, x \neq 3 \text{ and } \dfrac{x+2}{x-3} \neq -1\right\} = \{x \mid x \neq 3 \text{ and } x \neq 1/2\}$.