Math 125 Exam #4 Key (Fall 2022)

1 $\csc\theta > 0$ and $\cot\theta < 0$ implies θ is in Quadrant II, with $r = 3$, $y = 1$, and $x = -2\sqrt{2}$. Thus $\sin\theta = \frac{1}{3}$, $\cos\theta = -\frac{2\sqrt{2}}{3}$, $\tan\theta = -\frac{1}{2\sqrt{2}}$, $\cot\theta = -2\sqrt{2}$, $\sec\theta = -\frac{3}{2\sqrt{2}}$.

2a $\dfrac{3\pi}{10}$ **2b** $\dfrac{\pi}{7}$ **2c** $\sqrt{5}$ **2d** Undefined: $\sin\frac{7\pi}{6}$ is not in domain of $\sec^{-1}$.

3 $f^{-1}(x) = \frac{1}{3}\cos^{-1}(-x/2)$, with $D_{f^{-1}} = R_f = [-2, 2]$ and $R_{f^{-1}} = D_f = [0, \frac{\pi}{3}]$.

4a From $(\tan\theta)(3\tan^2\theta - 1) = 0$ we have $\tan\theta = 0$ (so $\theta = 0, \pi$), or $\tan\theta = \frac{1}{\sqrt{3}}$ (so $\theta = \frac{\pi}{6}, \frac{7\pi}{6}$), or $\tan\theta = -\frac{1}{\sqrt{3}}$ (so $\theta = \frac{5\pi}{6}, \frac{11\pi}{6}$). Solution set is $\left\{0, \pi, \frac{\pi}{6}, \frac{5\pi}{6}, \frac{7\pi}{6}, \frac{11\pi}{6}\right\}$.

4b Either $\sec\theta = -\sqrt{2}$ (so $\theta = \frac{3\pi}{4}, \frac{5\pi}{4}$) or $\sec\theta = \sqrt{2}$ (so $\theta = \frac{\pi}{4}, \frac{7\pi}{4}$). The solution set is: $\left\{\frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}\right\}$.

4c Write $4 + 4\sin\theta = 1 - \sin^2\theta$, so $(\sin\theta + 3)(\sin\theta + 1) = 0$. Since $\sin\theta = -3$ is impossible, we have only $\sin\theta = -1$, and thus $\theta = \frac{3\pi}{2}$.

5a Starting with the left-hand side, we have

$$\frac{\dfrac{1}{\sin} - \dfrac{\cos}{\sin}}{\dfrac{1}{\cos} - 1} \cdot \frac{\cos}{\cos} = \frac{\dfrac{(1-\cos)\cos}{\sin}}{1-\cos} = \frac{\cos}{\sin} = \cot.$$

5b Starting with the left-hand side,

$$\frac{1}{1-\sin} = \frac{1+\sin}{(1-\sin)(1+\sin)} = \frac{1+\sin}{1-\sin^2} = \frac{1+\sin}{\cos^2} = \frac{1}{\cos^2} + \frac{\sin}{\cos}\cdot\frac{1}{\cos} = \sec^2 + \tan\sec.$$

6 Let d be the distance the target is missed. Then $\tan 0.4^\circ = \dfrac{d}{384{,}000}$ implies

$$d = 384{,}000 \tan 0.4^\circ = 2683.7 \approx 2700 \text{ km}.$$

7 Let d be the distance between ship and lighthouse. Then

$$\tan 21^\circ = \frac{70}{d} \quad\Rightarrow\quad d = \frac{70}{\tan 21^\circ} = 182.4 \approx 182 \text{ m}.$$

8a $B = 110°$ is immediate, and with the Law of Sines we find that $b = 3.68$ and $c = 1.34$.

8b Use the Law of Sines to get $\sin A = 2\sin 100° \approx 1.97$, which is impossible, and so no triangle results.

9 Letting d be the distance, with the Law of Sines we have

$$\frac{\sin 7°}{d} = \frac{\sin 38°}{600\sqrt{2}},$$

and hence $d = 168.0$ meters.