Math 125 Exam #3 Key (Fall 2022)

1a We have

$$(f\circ g)(x)=f(g(x))=f(2/\sqrt{x})=\frac{2/\sqrt{x}}{1-4/\sqrt{x}}=\frac{2}{\sqrt{x}-4}$$

and

$$(g\circ f)(x)=g(f(x))=g\left(\frac{x}{1-2x}\right)=\frac{2}{\sqrt{\dfrac{x}{1-2x}}}.$$

1b $D_{f\circ g}=\{x:x\in D_g\ \&\ g(x)\in D_f\}$ with $D_g=(0,\infty)$ and $D_f=(-\infty,\frac{1}{2})\cup(\frac{1}{2},\infty)$. Thus:

$$D_{f\circ g}=\left\{x:x>0\ \ \&\ \ \frac{2}{\sqrt{x}}\neq\frac{1}{2}\right\}=(0,\tfrac{1}{16})\cup(\tfrac{1}{16},\infty).$$

1c $D_{g\circ f}=\{x:x\in D_f\ \&\ f(x)\in D_g\}$, so

$$D_{g\circ f}=\left\{x:x\neq\frac{1}{2}\ \ \&\ \ \frac{x}{1-2x}>0\right\}=(0,\tfrac{1}{2}).$$

2 Need $(f\circ g)(0)=-20$, where $(f\circ g)(0)=f(g(0))=3c^2-7$. Thus c must be such that $3c^2-7=-20$, or $c^2=-\frac{13}{3}$. But there is no real c value that can satisfy this, and so there is no solution here.

3a Let $y=T(x)$, so $y=\frac{2x-3}{x+4}$. Solve for x to get $x=\frac{4y+3}{2-y}$, and thus

$$T^{-1}(y)=\frac{4y+3}{2-y}.$$

3b $R_T=D_{T^{-1}}=(-\infty,2)\cup(2,\infty)$ and $R_{T^{-1}}=D_T=(-\infty,-4)\cup(-4,\infty)$.

4a $6^{2x+3}=6^{-3}$ implies $2x+3=-3$, so $x=-3$.

4b Write $x^3=\frac{1}{8}$, so $x=\frac{1}{2}$.

4c Write $\log_3\dfrac{(x+4)^2}{9}=2$, so $\dfrac{(x+4)^2}{9}=3^2$, giving $x=-13,5$. But -13 is extraneous, so $x=5$.

4d Write $\log_5(x+3)(x-1)=1$, so $(x+3)(x-1)=5$, giving $x=-4,2$. But -4 is extraneous, so $x=2$.

4e Write $(3^x)^2 - 3 \cdot 3^x + 1 = 0$. Letting $u = 3^x$ makes this $u^2 - 3u + 1 = 0$, so that

$$3^x = u = \frac{3 \pm \sqrt{3^2 - 4(1)(1)}}{2(1)} = \frac{3 \pm \sqrt{5}}{2},$$

and thus $x = \log_3 \frac{3\pm\sqrt{5}}{2}$.

4f With the change-of-base formula:

$$\log_2 x + \frac{\log_2 x}{\log_2 4} + \frac{\log_2 x}{\log_2 8} = 11 \quad \hookrightarrow \quad \log_2 x + \frac{\log_2 x}{2} + \frac{\log_2 x}{3} = 11 \quad \hookrightarrow \quad \log_2 x = 6,$$

so $x = 2^6 = 64$.

5a Set $y = \omega(x)$, so $y = 3 - 2\log_8(8 - x)$ and hence $x = 8 - 8^{(3-y)/2}$. Therefore we have $\omega^{-1}(y) = 8 - 8^{(3-y)/2}$.

5b $R_{\omega^{-1}} = D_\omega = (-\infty, 8)$ and $R_\omega = D_{\omega^{-1}} = (-\infty, \infty)$.

6 Given: $A_0 = A(0) = 0.25$ and $A(17) = 0.15$, and so $A(t) = 0.25e^{-kt}$. Now,

$$0.15 = A(17) = 0.25e^{-17k} \quad \Rightarrow \quad e^{-17k} = 0.6 \quad \Rightarrow \quad k = -\frac{\ln 0.6}{17} \approx 0.0300,$$

so that $A(t) = 0.25e^{-0.03t}$. From this we get $A(30) = 0.10$ mole (i.e. 0.10 mole remains after 30 minutes).

Solve $A(t) = 0.02$ to find the time 0.02 mole remains:

$$0.25e^{-0.03t} = 0.02 \quad \Rightarrow \quad e^{-0.03t} = 0.08 \quad \Rightarrow \quad t = -\frac{\ln 0.08}{0.03} \approx 84.2 \text{ minutes}.$$

7 Convert 0.444° to minutes: $(0.444^\circ)\left(\dfrac{60'}{1^\circ}\right) = 26.64'$.

Convert $0.64'$ to seconds, and round to the nearest second: $(0.64')\left(\dfrac{60''}{1'}\right) = 38.4'' \approx 38''$.

Therefore $44.444^\circ \approx 44^\circ\, 26'\, 38''$.

8 The point given lies on a circle of radius $r = 5$, so:

$$\sin\varphi = \frac{4}{5},\quad \cos\varphi = -\frac{3}{5},\quad \tan\varphi = -\frac{4}{3},\quad \csc\varphi = \frac{5}{4},\quad \sec\varphi = -\frac{5}{3},\quad \cot\varphi = -\frac{3}{4}.$$

9 With use of the 30-60-90 right triangle in the first quadrant we find the angle to be $\frac{\pi}{3}$. Other angles are given by $\frac{\pi}{3} + 2\pi n = \frac{\pi + 6\pi n}{3}$ for integers n, so letting $n = -2, -1, 0, 1, 2$ we obtain the angles

$$-\frac{11\pi}{3},\quad -\frac{5\pi}{3},\quad \frac{\pi}{3},\quad \frac{7\pi}{3},\quad \frac{13\pi}{3}.$$