Math 125 Exam #2 Key (Fall 2022)

1 Each cable forms a parabola with vertex at $(0,0)$ (the point where the cable touches the road) and points $(\pm 200, 75)$ (the tops of the towers). With $h = k = 0$ in the vertex form, the quadratic function is $f(x) = ax^2$. Now, $a(200)^2 = f(200) = 75$ (the height of a tower), so that $a = 0.001875$ and we have $f(x) = 0.001875x^2$. The height of either cable 100 m from the center is $f(\pm 100) = 0.001875(\pm 100)^2 = 18.75$ m.

2 Solve $f(x) = 0$, or $-2x^2 + 8x + 1 = 0$. With the quadratic forumula we obtain $x = 2 \pm \frac{3\sqrt{2}}{2}$.

3 Write $|x/2| = 2$, so $x/2 = \pm 2$ and finally $x = \pm 4$.

4 From $|1 - 4x| > 6$ we have $1 - 4x < -6$ or $1 - 4x > 6$, and so $x > \frac{7}{4}$ or $x < -\frac{5}{4}$. Solution set: $(-\infty, -\frac{5}{4}) \cup (\frac{7}{4}, \infty)$.

5 We have $f(x) = Cx^2(x-2)(x+1)^2$, with $4 = f(1) = C(-1)(2)^2$ implying that $C = -1$. Thus the polynomial function is $f(x) = -x^2(x-2)(x+1)^2$.

6 To have real coefficients the Conjugate Zeros Theorem implies that $2 - i$ must also be a zero, and so we need

$$\begin{aligned} f(x) &= (x+4)[x-(2+i)][x-(2-i)] \\ &= (x+4)(x^2-4x+5) \\ &= x^3 - 11x + 20. \end{aligned}$$

7 Let $f(x) = x^4 + 6x^3 + 7x^2 - 6x - 8$, so the problem is to find all x such that $f(x) = 0$. Among the possible rational zeros is 1, which turns out to work:

$$\begin{array}{r|rrrr|r} 1 & 1 & 6 & 7 & -6 & -8 \\ & & 1 & 7 & 14 & 8 \\ \hline & 1 & 7 & 14 & 8 & 0 \end{array} \quad \longrightarrow \quad f(x) = (x-1)(x^3 + 7x^2 + 14x + 8)$$

Now we do the same for $g(x) = x^3 + 7x^2 + 14x + 8$, which has -1 as a zero:

$$\begin{array}{r|rrr|r} -1 & 1 & 7 & 14 & 8 \\ & & -1 & -6 & -8 \\ \hline & 1 & 6 & 8 & 0 \end{array} \quad \longrightarrow \quad g(x) = (x+1)(x^2+6x+8) = (x+1)(x+2)(x+4)$$

Hence $f(x) = (x-1)(x+1)(x+2)(x+4)$ is the full factorization, and the equation's solution set is $\{1, -1, -2, -4\}$.

8 The vertical asymptotes are $x = \pm 2$, and with the division

$$\begin{array}{r|llll} & x & +3 & & \\ \hline x^2 - 4 & x^3 + 3x^2 & & & \\ & -\,x^3 & & +\,4x & \\ \hline & & 3x^2 + 4x & & \\ & & -\,3x^2 & & +\,12 \\ \hline & & & 4x + 12 & \end{array}$$

we find $y = x + 3$ to be an oblique asymptote.

9a Rewrite as $(x-2)(x+1) < 0$. Using the Intermediate Value Theorem we find the solution set to be $(-1, 2)$.

9b Rewrite as

$$\frac{x(3x+2)}{x+1} \leq 0.$$

The left-hand side has zeros $-\frac{2}{3}$ and 0, and vertical asymptote $x = -1$. Choose test values in the intervals $(-\infty, -1)$, $(-1, -\frac{2}{3})$, $(-\frac{2}{3}, 0)$, and $(0, \infty)$, and use the Intermediate Value Theorem to find the solution set to be $(-\infty, -1) \cup [-\frac{2}{3}, 0]$.

10 Find all x such that $x^3 \leq 8x^2$, which becomes $x^2(x-8) \leq 0$. Since $x^2 > 0$ for any $x \neq 0$, the inequality is satisfied if and only if $x \leq 8$. Solution set is $(-\infty, 8]$.