Math 125 Exam #4 Key (Fall 2021)

1 $\csc\theta > 0$ and $\cot\theta < 0$ implies θ is in Quadrant II, with $r = 3$, $y = 1$, and $x = -2\sqrt{2}$. Thus $\sin\theta = \frac{1}{3}$, $\cos\theta = -\frac{2\sqrt{2}}{3}$, $\tan\theta = -\frac{1}{2\sqrt{2}}$, $\cot\theta = -2\sqrt{2}$, $\sec\theta = -\frac{3}{2\sqrt{2}}$.

2 Since tan is an odd function, $f(-a) = -f(a) = -6$. Also tan has period π, so $f(a+\pi) + f(a-3\pi) = f(a) + f(a) = 12$.

3a $\frac{3\pi}{10}$ **3b** $\frac{\pi}{7}$ **3c** $\sqrt{5}$ **3d** Undefined: $\sin\frac{7\pi}{6}$ is not in domain of $\sec^{-1}$.

4 $f^{-1}(x) = \frac{1}{3}\cos^{-1}(-x/2)$, with $D_{f^{-1}} = R_f = [-2, 2]$ and $R_{f^{-1}} = D_f = [0, \frac{\pi}{3}]$.

5a We have $\cos\theta = \pm\frac{1}{\sqrt{2}}$, and so $\theta = \frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}$.

5b So $\tan\theta = -1$, giving $\theta = \frac{3\pi}{4}, \frac{7\pi}{4}$.

5c Write $4 + 4\sin\theta = 1 - \sin^2\theta$, so $(\sin\theta + 3)(\sin\theta + 1) = 0$, which implies $\sin\theta = -1$, or $\theta = \frac{3\pi}{2}$.

6a We have

$$1 - \frac{\sin^2\theta}{1-\cos\theta} = 1 - \frac{1-\cos^2\theta}{1-\cos\theta} = 1 - \frac{(1-\cos\theta)(1+\cos\theta)}{1-\cos\theta} = 1 - (1+\cos\theta) = -\cos\theta.$$

6b We have

$$\sin\theta\tan\theta = \frac{\sin^2\theta}{\cos\theta} = \frac{1-\cos^2\theta}{\cos\theta} = \frac{1}{\cos\theta} - \frac{\cos^2\theta}{\cos\theta} = \sec\theta - \cos\theta.$$

7 With a half-angle identity,

$$\tan 15^\circ = \frac{\sin 30^\circ}{1+\cos 30^\circ} = \frac{1/2}{1+\sqrt{3}/2} = 2 - \sqrt{3}.$$

8 This sets up a right triangle with legs of length 16 and 3, and so if the angle of depression is θ, we have

$$\tan\theta = -\frac{3}{16} \quad\Rightarrow\quad \theta = \tan^{-1}(-0.1875) = -10.6^\circ.$$

9 Let h be the height of the monument. Then

$$\tan 35.1^\circ = \frac{h}{789} \quad\Rightarrow\quad h = 789\tan 35.1^\circ \approx 554.5 \text{ ft}$$

10a $B = 110^\circ$ is immediate, and with the Law of Sines we find that $b = 3.68$ and $c = 1.34$.

10b Use the Law of Sines to get $\sin A = 2\sin 100^\circ \approx 1.97$, which is impossible, and so no triangle results.

11 This is the ambiguous case of the Law of Sines. The two possible distances to Venus are 164,200,000 km and 65,000,000 km.