Math 125 Exam #4 Key (Fall 2020)

1a $\pi/3$ (the work of drawing the appropriate triangle and circle is omitted)

1b $2\pi/3$ (the work of drawing the appropriate triangle and circle is omitted)

2a Setting $y = f(x)$, we solve for x to obtain $f^{-1}(y) = \tan^{-1}\left(\dfrac{y+3}{2}\right)$.

2b Ran $f = (-\infty, \infty)$, Dom $f^{-1} = (-\infty, \infty)$, Ran $f^{-1} = (-\frac{\pi}{2}, \frac{\pi}{2})$.

3a Factor: $(2\cos\theta - 1)(\cos\theta + 1) = 0$, so either $\cos\theta = \frac{1}{2}$ or $\cos\theta = -1$. Solution set: $\{\pi, \frac{\pi}{3}, \frac{5\pi}{3}\}$.

3b Either $\cot\theta = -1$ or $\csc\theta = \frac{1}{2}$. The latter equation has no solution, but the former gives the solution set $\{\frac{3\pi}{4}, \frac{7\pi}{4}\}$.

4a Get a common denominator on the left-hand side, giving:

$$\frac{\cos^2 v + (1+\sin v)^2}{\cos v(1+\sin v)} = \frac{2 + 2\sin v}{\cos v(1+\sin v)} = \frac{2}{\cos v} = 2\sec v.$$

4b The left-hand side LHS is:

$$\text{LHS} = \frac{\sin\theta}{\cos\theta} + \frac{\cos\theta}{\sin\theta} = \frac{\sin^2\theta + \cos^2\theta}{\cos\theta\sin\theta} = \frac{1}{\cos\theta\sin\theta} = \sec\theta\csc\theta.$$

5 With the given information we find that $\sin\alpha = \sqrt{3}/2$ and $\cos\beta = 2\sqrt{2}/3$. Now,

$$\sin(\alpha+\beta) = \sin\alpha\cos\beta + \cos\alpha\sin\beta = \frac{\sqrt{6}}{3} + \frac{1}{6},$$

and

$$\tan(\alpha-\beta) = \frac{\tan\alpha - \tan\beta}{1 + \tan\alpha\tan\beta} = \frac{-\sqrt{3} - \frac{1}{2\sqrt{2}}}{1 - \sqrt{3}\cdot\frac{1}{2\sqrt{2}}} = \frac{2\sqrt{6}+1}{\sqrt{3} - 2\sqrt{2}} = -\frac{8\sqrt{2}+9\sqrt{3}}{5}.$$

6a Work with the left-hand side:

$$\text{LHS} = \frac{1}{\cos(\alpha-\beta)} = \frac{1}{\cos\alpha\cos\beta + \sin\alpha\sin\beta}\cdot\frac{\sec\alpha\sec\beta}{\sec\alpha\sec\beta} = \text{RHS}.$$

6b $\tan\frac{v}{2} = \frac{\sin v}{1+\cos v} = \frac{\sin v(1-\cos v)}{1-\cos^2 v} = \frac{\sin v(1-\cos v)}{\sin^2 v} = \frac{1-\cos v}{\sin v} = \csc v - \cot v.$

7 Equation becomes $2\sin\theta\cos\theta = \cos\theta$, and so either $\cos\theta = 0$ or $\sin\theta = \frac{1}{2}$. Solution set: $\{\frac{\pi}{2}, \frac{3\pi}{2}, \frac{\pi}{6}, \frac{5\pi}{6}\}$.

8 Height is $22\sin 70° \approx 20.7$ ft.

9a Use Law of Sines: $b = 0.65$, $C = 100°$, $c = 1.29$.

9b Use Law of Sines to find two triangles: $B_1 = 13.4°$, $C_1 = 156.6°$, $c_1 = 6.86$ and $B_2 = 166.6°$, $C_2 = 3.4°$, $c_2 = 1.02$.

9c Requires Law of Cosines, starting with

$$A = \cos^{-1}\left(\frac{a^2 - b^2 - c^2}{-2bc}\right) = \cos^{-1}(0.1161) = 83.3°.$$

Use the law again to get $B = 44.1°$, and thus $C = 52.6°$.

10 We have a triangle with base 100, and two angles of $B = 40°$ and $C = 25°$ with the base. The angle opposite the base is thus $A = 115°$, and with the Law of Sines we find the side opposite the angle $B = 40°$ is $b = 70$. Finally, the height of the helicopter is

$$h = b\sin C = 70\sin 25° = 29.6 \approx 30 \text{ m}.$$