Math 125 Exam #3 Key (Fall 2020)

1 Domain is all x for which $(2x+3)/(x-1) > 0$, occurring if $2x+3>0$ and $x-1>0$, or if $2x+3<0$ and $x-1<0$. The first case has solution $x>1$, the second case has solution $x<-\frac{3}{2}$. The domain is therefore $(-\infty,-\frac{3}{2})\cup(1,\infty)$.

2a Let $y=f(x)$, so $y=1-6\log_4(3-x)$, and hence $4^{(1-y)/6}=3-x$. Solving for x gives $x=3-4^{(1-y)/6}$, and therefore

$$f^{-1}(y)=3-4^{(1-y)/6}.$$

2b Ran $f^{-1}=$ Dom $f=(-\infty,3)$ and Ran $f=$ Dom $f^{-1}=(-\infty,\infty)$.

3 By the LAWS OF LOGARITHMS, the expression becomes

$$\log_b(2x+4)^3-\log_b(1-2x)^2-\log_b(2x)=\log_b\frac{(2x+4)^3}{2x(1-2x)^2}.$$

4 Let $y=f(x)$. Then $y=\log_a x$, whence

$$a^y=x \;\Rightarrow\; (1/a)^{-y}=x \;\Rightarrow\; \log_{1/a}(1/a)^{-y}=\log_{1/a}x \;\Rightarrow\; -y=\log_{1/a}x,$$

and therefore $-f(x)=\log_{1/a}x$.

5a We get $\ln\left(\dfrac{x+1}{x}\right)=2$, so $e^2=\dfrac{x+1}{x}$ and thus $x=(e^2-1)^{-1}$.

5b Rewrite as $\log_2\left(\dfrac{x}{25}\right)=\log_2\left(\dfrac{x+1}{100}\right)$, so $\dfrac{x}{25}=\dfrac{x+1}{100}$ and hence $x=\frac{1}{3}$.

5c Let $u=\log_3 x$, so equation becomes $u^2-5u-6=0$, which has solutions $u=-1,6$. So $\log_3 x=-1$ or $\log_3 x=6$, giving solutions $x=\frac{1}{3}$ and $x=3^6=729$.

5d Write equation as $(3^x)^2-3\cdot3^x+1=0$. Let $u=3^x$ so equation becomes $u^2-3u+1=0$, and solve this to get $u=\frac{3\pm\sqrt{5}}{2}$. Thus $3^x=\frac{3\pm\sqrt{5}}{2}$, and so $x=\log_3\left(\frac{3\pm\sqrt{5}}{2}\right)$.

5e With the Change-of-Base Formula we get

$$\log_2(3x+2)-\frac{\log_2 x}{\log_2 4}=3 \;\Rightarrow\; \log_2(3x+2)-\frac{1}{2}\log_2 x=3 \;\Rightarrow\; \log_2\frac{3x+2}{\sqrt{x}}=3,$$

and thus

$$\frac{3x+2}{\sqrt{x}} = 2^3 \quad \Rightarrow \quad 3(\sqrt{x}\,)^2 - 8\sqrt{x} + 2 = 0 \quad \Rightarrow \quad \sqrt{x} = \frac{4 \pm \sqrt{10}}{3}.$$

Solution set is $\left\{ \left(\frac{4 \pm \sqrt{10}}{3} \right)^2 \right\}$.

6a $N(t) = N_0 e^{kt}$.

6b Let $t = 0$ for the year 2004, so $N_0 = N(0) = 900{,}000$, and then $N(t) = 900{,}000e^{kt}$. Using $N(4) = 800{,}000$ gives

$$800{,}000 = 900{,}000e^{4k} \quad \Rightarrow \quad k = \tfrac{1}{4} \ln \tfrac{8}{9},$$

so

$$N(t) = 900{,}000e^{(t/4)\ln(8/9)} = 900{,}000 \left(\frac{8}{9} \right)^{t/4}.$$

Population in 2012 is thus

$$N(8) = 900{,}000 \left(\frac{8}{9} \right)^2 \approx 711{,}111.$$

7 We're given $u_0 = 100\,°\text{C}$ (so $u(0) = 100$), $T = 22\,°\text{C}$, and also $u(30) = 80$. With Newton's Law of Cooling:

$$80 = u(30) = 22 + (100 - 22)e^{30k},$$

which gives $k = \frac{1}{30} \ln \frac{58}{78} \approx -0.00988$. Thus the model is

$$u(t) = 22 + 78e^{-0.00988t}.$$

After another 30 minutes (when $t = 60$), the temperature is

$$u(60) = 22 + 78e^{-0.00988(60)} \approx 65.1\,°\text{C}.$$

8 Convert $0.329°$ to minutes:

$$(0.329°) \left(\frac{60'}{1°} \right) = 19.74'.$$

Convert $0.74'$ to seconds, and round to the nearest second:

$$(0.74') \left(\frac{60''}{1'} \right) = 44.4'' \approx 44''.$$

Therefore $107.329° \approx 107°\,19'\,44''$.

9 The point given lies on a circle of radius $\sqrt{5}$, so:

$$\sin\theta = -\frac{2}{\sqrt{5}}, \quad \cos\theta = -\frac{1}{\sqrt{5}}, \quad \tan\theta = 2, \quad \csc\theta = -\frac{\sqrt{5}}{2}, \quad \sec\theta = -\sqrt{5}, \quad \cot\theta = \frac{1}{2}.$$

10 We need only use fundamental trigonometric identities to find that

$$\tan\theta = \frac{\sin\theta}{\cos\theta} = \sqrt{3}, \quad \cot\theta = \frac{1}{\tan\theta} = \frac{1}{\sqrt{3}}, \quad \csc\theta = \frac{1}{\sin\theta} = \frac{2}{\sqrt{3}}, \quad \sec\theta = \frac{1}{\cos\theta} = 2.$$

11 Right off we have $\tan\theta = \frac{3}{4}$. Now, $\cos\theta < 0$ implies θ is in quadrant QII or QIII, while $\cot\theta < 0$ implies θ is in QI or QIII. Thus θ is in QIII, and since $\sec\theta < 0$ in QIII, we have

$$\sec^2\theta = 1 + \tan^2\theta \quad \Rightarrow \quad \sec\theta = -\sqrt{1+\tan^2\theta} = -\sqrt{1+(3/4)^2} = -\frac{5}{4}.$$

The rest come easily:

$$\cos\theta = -\frac{4}{5}, \quad \sin\theta = \cos\theta\tan\theta = -\frac{4}{5}\cdot\frac{3}{4} = -\frac{3}{5}, \quad \csc\theta = -\frac{5}{3}.$$