Math 125 Exam #2 Key (Fall 2020)

1 $-3i$ must be another zero, and so $(x-3i)(x+3i) = x^2+9$ is a factor of $f(x)$. Then

$$
\begin{array}{r|rrrrr}
 & & & 3x^2 & +5x & -2 \\
\hline
x^2+9 & 3x^4 & +5x^3 & +25x^2 & +45x & -18 \\
 & -3x^4 & & -27x^2 & & \\
\cline{2-4}
 & & 5x^3 & -2x^2 & +45x & \\
 & & -5x^3 & & -45x & \\
\cline{3-5}
 & & & -2x^2 & & -18 \\
 & & & 2x^2 & & +18 \\
\cline{4-6}
 & & & & & 0
\end{array}
$$

shows $3x^2+5x-2$ is another factor. Now,

$$f(x) = 0 \quad\Rightarrow\quad (x^2+9)(3x^2+5x-2) = 0 \quad\Rightarrow\quad (x-3i)(x+3i)(3x-1)(x+2) = 0,$$

and therefore other zeros of f are $\frac{1}{3}$ and -2.

2 Possible rational zeros are $\pm1, \pm2, \pm\frac{1}{2}, \pm\frac{1}{4}$. Synthetic division shows -2 to be a zero:

$$
\begin{array}{r|rrr|r}
-2 & 4 & 4 & -7 & 2 \\
 & & 2 & 2 & -8 \\
\hline
 & 4 & -4 & 1 & 0
\end{array}
\quad\longrightarrow\quad 4x^2-4x+1.
$$

So $g(x) = (x+2)(4x^2-4x+1) = (x+2)(2x-1)^2$ and the zeros of g are -2, $\frac{1}{2}$ (multiplicity 2).

3 -2 will need to be a zero for h, which by the Remainder Theorem implies that the following division should have remainder 0:

$$
\begin{array}{r|rrrr|r}
-2 & 1 & -k & k & 0 & 1 \\
 & & -2 & 2k+4 & -6k-8 & 12k+16 \\
\hline
 & 1 & -k-2 & 3k+4 & -6k-8 & 12k+17
\end{array}
$$

Thus we need $12k+17=0$, or $k = -\frac{17}{12}$.

4 There vertical asymptote $x = -\frac{1}{4}$, and from the division

$$
\begin{array}{r|rrr}
 & & 2x & +6 \\
\hline
4x+1 & 8x^2 & +26x & -7 \\
 & -8x^2 & -2x & \\
\cline{2-3}
 & & 24x & -7 \\
 & & -24x & -6 \\
\cline{3-4}
 & & & -13
\end{array}
$$

we conclude that $y = 2x+6$ is an oblique asymptote.

5a Write $x^3 + x^2 - 4x - 4 < 0$, and then $(x+1)(x-2)(x+2) < 0$ (factor by grouping). Using test points and the Intermediate Value Theorem (work omitted here) we find the solution set to be $(-\infty, -2) \cup (-1, 2)$.

5b Solution set is $(-\infty, -1) \cup (3, \infty)$.

5c Solution set is $(-\infty, 1) \cup [2, \infty)$.

6a $(g \circ g)(-2) = 26$ and $(f \circ f)(82) = \sqrt{8} = 2\sqrt{2}$.

6b $(f \circ g)(x) = f(g(x)) = f(1 + x^2) = \sqrt{(1 + x^2) - 1} = \sqrt{x^2} = |x|$.

$$\begin{aligned}\mathrm{Dom}(f \circ g) &= \big\{x : x \in \mathrm{Dom}\, g \ \ \& \ \ g(x) \in \mathrm{Dom}\, f\big\}\\ &= \{x : x \in (-\infty, \infty) \ \ \& \ \ 1 + x^2 \geq 1\}\\ &= \{x : x^2 \geq 0\} = (-\infty, \infty).\end{aligned}$$

6c $(g \circ f)(x) = g(f(x)) = g(\sqrt{x - 1}) = 1 + (\sqrt{x - 1})^2 = 1 + (x - 1) = x$. However, by the definition of the domain of a composition of functions we have $\mathrm{Dom}(g \circ f) = [1, \infty)$.

6d $(f \circ f)(x) = f(f(x)) = \sqrt{\sqrt{x - 1} - 1}$, with domain $[2, \infty)$.

7 We have $(f \circ g)(x) = 12x^2 + 12cx + 3c^2 - 7$, so that $(f \circ g)(0) = 3c^2 - 7$, and we need $(f \circ g)(0) = 68$. So c must be such that $3c^2 - 7 = 68$, or $c = \pm 5$.

8a Let $y = F(x)$. Then

$$y = \frac{2x - 3}{x + 4} \quad \Leftrightarrow \quad xy + 4y = 2x - 3 \quad \Leftrightarrow \quad x = \frac{4y + 3}{2 - y},$$

and since $y = F(x)$ if and only if $x = F^{-1}(y)$, we now have

$$F^{-1}(y) = \frac{4y + 3}{2 - y},$$

or equivalently

$$F^{-1}(x) = \frac{4x + 3}{2 - x}.$$

8b $\mathrm{Ran}\, F = \mathrm{Dom}\, F^{-1} = (-\infty, 2) \cup (2, \infty)$ and $\mathrm{Ran}\, F^{-1} = \mathrm{Dom}\, F = (-\infty, -4) \cup (-4, \infty)$.