Math 125 Exam #1 Key (Fall 2020)

1a $\text{Dom}(q) = \{x : x^3 + 4x^2 \neq 0\} = (-\infty, -4) \cup (-4, 0) \cup (0, \infty)$.

1b $\text{Dom}(v) = \{t : -t - 5 > 0\} = (-\infty, -5)$.

2a We have

$$(f - g)(x) = \frac{3}{2x} - \frac{x+1}{x-1},$$

with

$$\begin{aligned}\text{Dom}(f - g) &= \text{Dom}(f) \cap \text{Dom}(g) = \{x : x \neq 0\} \cap \{x : x \neq 1\} \\ &= (-\infty, 0) \cup (0, 1) \cup (1, \infty).\end{aligned}$$

2b We have

$$(f/g)(x) = \frac{3}{2x} \div \frac{x+1}{x-1} = \frac{3(x-1)}{2x(x+1)},$$

with domain the same as $\text{Dom}(f - g)$ except we must also exclude x such that $g(x) = 0$ (i.e. $x = -1$). Thus

$$\text{Dom}(f/g) = (-\infty, -1) \cup (-1, 0) \cup (0, 1) \cup (1, \infty).$$

3 We have

$$16 = f(-1) = 3(-1)^2 - C(-1) + 9 = C + 12 \quad \Rightarrow \quad C = 4.$$

4a $\text{Dom}(F) = [-4, 5)$ and $\text{Ran}(F) = [-2, 2] \cup \{3\}$.

4b Definition for F:

$$f(x) = \begin{cases} -2x - 6, & -4 \leq x \leq -2 \\ x, & -2 < x < 0 \\ 3, & 0 \leq x < 5 \end{cases}$$

5 Solve $f(x) = g(x)$, or $x^2 + 2x - 24 = 0$. Factoring yields $(x+6)(x-4) = 0$, giving $x = -6, 4$, and hence the intersection points $(-6, f(-6)) = (-6, 3)$ and $(4, f(4)) = (4, 33)$.

6 Vertex is at $(-b/2a, f(-b/2a)) = (-\frac{5}{4}, -\frac{1}{8})$. Range of f is $[-\frac{1}{8}, \infty)$.

7 Let x and y be the length and width of the rectangle, so $2x+2y = 200$, and thus $y = 100-x$. Now the area xy of the rectangle is, as a function of x, found to be

$$A(x) = x(100 - x) = -x^2 + 100x.$$

The vertex of this quadratic function is at $x = -b/2a = 50$ (where $a = -1$ and $b = 100$). The vertex is the maximum point on the parabola, and hence the area is maximized when $x = 50$ and $y = 100 - 50 = 50$. That is, a $50\,\mathrm{m} \times 50\,\mathrm{m}$ square yields the maximum area.

8 With the quadratic formula, $P(x) = 0$ implies $x = \dfrac{-6 \pm \sqrt{32}}{2} = -3 \pm 2\sqrt{2}$. These are the complex zeros of P. (Recall that real numbers are considered a subset of the complex number system.)

9 Either $x^2 + x = 12$ or $x^2 + x = -12$. The first equation gives $x = -4, 3$, while the second gives the complex numbers $-\frac{1}{2} \pm i\frac{\sqrt{47}}{2}$. Only the real solutions are required here, with extra credit given if the nonreal solutions are obtained.

10 Isolate the absolute value term to get

$$|4x - 1| < 7 \quad \Rightarrow \quad -7 < 4x - 1 < 7 \quad \Rightarrow \quad -\frac{3}{2} < x < 2,$$

and so $(-\frac{3}{2}, 2)$ is the solution set.