Math 125 Exam #4 Key (Fall 2019)

1a $-\pi/5$

1b Undefined.

1c $\sqrt{5}/2$

1d We have

$$\theta = \cos^{-1}\left(\sin\tfrac{7\pi}{6}\right) \quad\Rightarrow\quad \cos\theta = \sin\tfrac{7\pi}{6} = \cos\left(\tfrac{\pi}{2} - \tfrac{7\pi}{6}\right) = \cos\tfrac{2\pi}{3} \quad\Rightarrow\quad \theta = \tfrac{2\pi}{3}.$$

2a We have $\cos\theta = \pm\frac{1}{\sqrt{2}}$, and so $\theta = \frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}$.

2b So $\tan\theta = -1$, giving $\theta = \frac{3\pi}{4}, \frac{7\pi}{4}$.

2c Write $4 + 4\sin\theta = 1 - \sin^2\theta$, so $(\sin\theta + 3)(\sin\theta + 1) = 0$, which implies $\sin\theta = -1$, or $\theta = \frac{3\pi}{2}$.

3a We have

$$1 - \frac{\sin^2\theta}{1-\cos\theta} = 1 - \frac{1-\cos^2\theta}{1-\cos\theta} = 1 - \frac{(1-\cos\theta)(1+\cos\theta)}{1-\cos\theta} = 1 - (1+\cos\theta) = -\cos\theta.$$

3b We have

$$\sin\theta\tan\theta = \frac{\sin^2\theta}{\cos\theta} = \frac{1-\cos^2\theta}{\cos\theta} = \frac{1}{\cos\theta} - \frac{\cos^2\theta}{\cos\theta} = \sec\theta - \cos\theta.$$

4 The expression becomes $\cos\left(\frac{5\pi}{12} + \frac{7\pi}{12}\right) = \cos\pi = -1$.

5 With a half-angle identity,

$$\tan 15^\circ = \frac{\sin 30^\circ}{1 + \cos 30^\circ} = \frac{1/2}{1 + \sqrt{3}/2} = 2 - \sqrt{3}.$$

6 $\cos^4\theta - \sin^4\theta = (\cos^2\theta - \sin^2\theta)(\cos^2\theta + \sin^2\theta) = \cos^2\theta - \sin^2\theta = \cos(2\theta)$.

7 This sets up a right triangle with legs of length 16 and 3, and so if the angle of depression is θ, we have

$$\tan\theta = -\frac{3}{16} \quad \Rightarrow \quad \theta = \tan^{-1}(-0.1875) = -10.6^\circ.$$

8 Let h be the height of the monument. Then

$$\tan 35.1^\circ = \frac{h}{789} \quad \Rightarrow \quad h = 789\tan 35.1^\circ \approx 554.5 \text{ ft}$$

9a $B = 110^\circ$ is immediate, and with the Law of Sines we find that $b = 3.68$ and $c = 1.34$.

9b Use the Law of Sines to get

$$\sin C = \frac{c\sin 40^\circ}{3} = 1.071.$$

Since there is no real value for C such that $\sin C > 1$, we conclude that there is no solution.

9c Use Law of Cosines to get

$$A = \cos^{-1}\frac{b^2 + c^2 - a^2}{2bc} = \cos^{-1}(0.9531) = 17.62^\circ.$$

Then, again with the Law of Cosines,

$$B = \cos^{-1}\frac{a^2 + c^2 - b^2}{2ac} = \cos^{-1}(0.8750) = 28.96^\circ.$$

To the tenths place we have $A = 17.6^\circ$, $B = 29.0^\circ$, and $C = 133.4^\circ$.

10 Use the Law of Cosines to find that the guy wires should be 520.1 ft and 499.5 ft.