Math 125 Exam #1 Key (Fall 2019)

1 $f(-4) = -39$ and $f(-x) = -2x^2 - x - 3$.

2a Domain of g is $\{x : 4 - 4x^2 \neq 0\} = \{x : x \neq \pm 1\}$.

2b Domain of h is $\{t : -t - 3 \geq 0 \text{ and } 3t - 15 \neq 0\} = (-\infty, -3]$

3a We have

$$(f \cdot g)(x) = \left(1 + \frac{1}{2x}\right)\left(\frac{1}{x-6}\right),$$

with domain $\{x : x \neq 0, 6\}$.

3b We have

$$(f/g)(x) = \frac{1 + \frac{1}{2x}}{\frac{1}{x-6}} = \frac{(2x+1)(x-6)}{2x}$$

with domain $\{x : x \neq 0, 6\}$.

4a Since $f(-x) = -3(-x)^2 - 5 = -3x^2 - 5 = f(x)$, f is even.

4b Since $g(-x) = \dfrac{-(-x)^3}{3(-x)^2 - 9} = \dfrac{x^3}{3x^2 - 9} = -g(x)$, g is odd.

5 Definition of f:

$$f(x) = \begin{cases} -\frac{5}{2}x - 7, & x \in [-4, -2] \\ x, & x \in (-2, 0] \\ -2x + 4, & x \in (0, 4) \end{cases}$$

6a Cut the wire into lengths $3x$ and $10 - 3x$. With the $3x$ piece made an equilateral triangle with sides of length x, and with the $10 - 3x$ piece make a circle. The area of the triangle is $\frac{\sqrt{3}}{4}x^2$, and the area of the circle (which has circumference $10 - 3x$) is $(10 - 3x)^2/4\pi$. Total area is

$$A(x) = \frac{\sqrt{3}}{4}x^2 + \frac{(10 - 3x)^2}{4\pi}.$$

6b Domain of A is $\{x : 0 < 3x < 10\}$, or $\left(0, \frac{10}{3}\right)$.

7 Solve $F(x) = 0$, or $2x^2 - 3x - 1 = 0$, using the quadratic formula or completing the square to get the zeros $(3 \pm \sqrt{17})/4$, which are also the x-intercepts.

8 Solve $h(x) = 0$, which becomes

$$x^2 + x + \tfrac{1}{2} = 0 \;\Rightarrow\; (x^2 + x + \tfrac{1}{4}) = -\tfrac{1}{4} \;\Rightarrow\; (x + \tfrac{1}{2})^2 = -\tfrac{1}{4} \;\Rightarrow\; x + \tfrac{1}{2} = \pm\tfrac{i}{2},$$

and thus $x = -\frac{1}{2} \pm \frac{1}{2}i$.

9 We have $x^2 - 7x - 8 > 0$, or $(x-8)(x+1) > 0$, which is satisfied if $x > 8$ or $x < -1$. Solution set: $(-\infty, -1) \cup (8, \infty)$.

10a With length x and width y the perimeter of the rectangle is $2x + 2y = 3000$, so $y = 1500 - x$. Area of rectangle is thus

$$A(x) = xy = x(1500 - x) = -x^2 + 1500x = -(x - 750)^2 + 750^2.$$

10b Area is greatest when $x = 750$.

10c Maximum area is $A(750) = 750^2$.

11 Write $|1 - 3t| = 4$, so either $1 - 3t = 4$ or $1 - 3t = -4$. Solution set: $\{-1, \frac{5}{3}\}$.

12 Write $|y-1| > 25$, so either $y-1 < -25$ or $y-1 > 25$. Solution set: $(-\infty, -24) \cup (26, \infty)$.